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A new simple and powerful normality test for progressively Type-II censored data

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Abstract: In this paper, a new goodness-of-fit test for a location-scale family based on progressively Type-II censored order statistics is proposed. Using Monte Carlo simulation studies, the present researchers have observed that the proposed test for normality is consistent and quite powerful in comparison with some existing goodness-of-fit tests based on progressively Type-II censored data. Also, the new test statistic for a real data set is used and the results show that the new proposed test statistic performs well.

Keywords: Goodness-of-fit testing; location-scale family; Monte Carlo simulation; order statistics; progressive Type-II censoring, spacings.
Mathematics Subject Classification (2010): 62F03, 62F10.

1 Introduction

One of the most interesting problems in statistics is finding a distribution which fits to a given data set. In other words, it is desired to test whether a specific distribution coincides with given data or not. To review the classical goodness-of-fit test problem, let $X_1, ..., X_n$ be random sample from an absolutely continuous population with cumulative distribution function (CDF) $F(.)$, and probability density function (PDF) $f(.)$. Based on the observed sample $x_1, ..., x_n$, an interesting test is

$$\begin{cases} H_0: & f = f_0 \\ H_1: & f \neq f_0, \end{cases} \tag{1}$$

which $f_0(x) = f_0(x; \boldsymbol{\theta})$, where $\boldsymbol{\theta} \in \Theta \subseteq \mathbb{R}^k$ is a k-vector parameter for some $k \in \mathbb{N}$. For more details on this topic, one may refer to D'Agostino and Stephens (1986) and Huber-Carol et al. (2002).

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The most of goodness-of-fit tests are built based on the distance between empirical distribution function (EDF) and theoretical distribution functions over the interval $(0, 1)$. The null hypothesis is rejected if the distance is too large in some metrics. However, one of the approaches for building a goodness-of-fit test is the deviation of each order statistic of uniform distribution from its expected value.

The classical goodness-of-fit tests for complete data is not usable for progressively Type-II censored data. Pakyari and Balakrishnan (2013) proposed a modification of the statistics based on order statistics and spacings including the class of C statistics, Greenwood's (1964) statistic, Quesenberry and Miller's (1975) statistic and Moran's (1951) statistic, to made them suitable for progressively Type-II censored data. Also, Torabi (2006, 2008) has introduced a new and general method for estimation and hypotheses testing using spacing.

For more details on the progressive Type-II censoring, one may refer to Balakrishnan (2007) and Balakrishnan and Aggarwala (2000). In progressive Type-II censoring, it is assumed that the removals of units are carried out at observed failure times and that the censoring scheme $(r_1, r_2, ..., r_m)$ is known in advance. Moreover, the number of units (n) and the number of observed failure times (m) are prefixed. Starting all n units at the same time, the first progressive censoring step takes place at the observation of the first failure time $X_{1:m:n}$, at this time, r_1 units are randomly chosen from the remained units and withdrawn from the experiment. Then, the experiment continues with the sample size $n - r_1 - 1$. After observing the next failure at time $X_{2:m:n}$, r_2 units are randomly removed from $n - r_1 - 2$ remained units. This process continued until the mth failure is observed. Then, the experiment ends. The failure times $X_{1:m:n}, ..., X_{m:m:n}$ are called progressive Type-II censored order statistics and $x_{1:m:n}, ..., x_{m:m:n}$ are the corresponding observations.

An exponentiality goodness-of-fit test based on spacings for progressively Type-II censored data, was proposed by Balakrishnan et al. (2002). Afterwards, they (2004) extended their method to general location–scale family of distributions. Also Wang (2008) proposed another goodness-of-fit test for the exponential distribution under progressively Type-II censored samples. Recently, Pakyari and Balakrishnan (2012) proposed a modification to the EDF goodness-of-fit statistics under progressively Type-II censored data. An exponentiality test based on Kullback–Leibler information for progressively Type-II censored data was proposed by Balakrishnan et al. (2012). Recently, Nadeb and Torabi (2018) have proposed a goodness-of-fit test statistic under progressive Type-II censoring from a location-scale distribution.

In Section 2, the test statistics based on spacings that were proposed by Pakyari and Balakrishnan (2013) are reviewed. These statistics are modification to the Greenwood's statistic, modification to the Quesenberry and Miller's statistic and modification to Moran's statistic. In Section 3, a new test statistic is proposed that will be used for test of normality under the progressively Type-II censored data. Also, consistency of this statistic is investigated using Monte Carlo simulation. The power of the proposed test is then assessed through Monte Carlo simulations in Section 4, and its performance is compared with those of the test procedures introduced earlier by Balakrishnan et al. (2004) and Pakyari and Balakrishnan (2013). It is shown that the proposed goodness-of-fit test is more powerful than or at least as good as the Balakrishnan et al.'s (2004) test and Pakyari and Balakrishnan's (2013) tests for different choices of sample sizes and progressive censoring schemes. In Section 5, we illustrate the application of proposed test procedure with a real data set.

2 Review on the test statistics based on spacings

Pakyari and Balakrishnan (2013) proposed the following statistics for goodness-of-fit test under progressively Type-II censored data:

$$C^+_{m:n} = \max_{1\leq i\leq m}(V_{i:m:n}), C^-_{m:n} = \max_{1\leq i\leq m}(-V_{i:m:n}), C_{m:n} = \max(C^-_{m:n}, C^+_{m:n}),$$

$$K_{m:n} = C^-_{m:n} + C^+_{m;n}, T^{(1)}_{m:n} = \sum_{i=1}^{m} \frac{V^2_{i:m:n}}{m}, T^{(2)}_{m:n} = \sum_{i=1}^{m} \frac{|V_{i:m:n}|}{m},$$

which, $V_{i:m:n} = U_{i:m:n} - \mu_{i:m:n}$, and $U_{i:m:n}$ is the ith order statistic from uniform (0,1) distribution under Type-II progressive censoring and $\mu_{i:m:n}$ is its expected value, i.e

$$\mu_{i:m:n} = 1 - \prod_{k=m-i+1}^{m} \frac{k + \sum_{j=m-i+1}^{m} r_j}{1 + k + \sum_{j=m-i+1}^{m} r_j}, \quad i = 1, \ldots, m.$$

It is easy to show that the distributions of all the above statistics do not depend on the location and scale parameters under location-scale transformations. Note that the family of densities $\{g(.;\mu,\sigma) : \mu \in \mathbb{R}, \sigma > 0\}$ is said to belong to location-scale family, if it is of the form $g(x;\mu,\sigma) = \frac{1}{\sigma}g_0(\frac{x-\mu}{\sigma})$; which $g_0(.)$ is a baseline density function and μ and σ are said to be location and scale parameters, respectively.

If the null hypothesis is true, we expect that $V_{i:m:n}$ to be small and consequently the above test statistics to be small. If the above test statistics exceed the corresponding upper-tail null critical values, the null hypothesis may be rejected. Recently, several goodness-of-fit statistics based on spacings have been developed. The one-step spacings are defined by

$$S_i = (n - r_1 - r_2 - ... - r_{i-1} - i + 1)(U_{i:m:n} - U_{i-1:m:n}), \quad i = 1, 2, ...m,$$

where $U_{0:m:n} = 0$.

The following statistics are based on the spacings that generalized by Pakyari and Balakrishnan (2013) under the progressively Type-II censored data:

- Statistic based on the sum of squares of the spacings of the form

$$G_{m:n} = \sum_{i=1}^{m} S_i^2.$$

- The generalization of Quesenberry and Miller's statistic for progressively Type-II censored samples will be of the form

$$Q_{m:n} = \sum_{i=1}^{m} S_i^2 + \sum_{i=1}^{m-1} S_i S_{i+1}.$$

The exact distributions of $G_{m:n}$ and $Q_{m:n}$ are not available explicitly but by Monte Carlo simulations the percentage points will be determined.

- The above statistics can also be defined in terms of higher order spacings. The overlapping k-step spacings, for integer k, are defined as

$$S_i^{(k)} = (n - r_1 - r_2 - ... - r_{i-1} - i + 1)(U_{i+k-1:m:n} - U_{i-1:m:n}), \quad i = 1, 2, ...m,$$

which $U_{l:m:n} = 1$ for $l > m$. Hartley and Pfaffenberger (1972) presented that the higher order spacings are useful for testing large complete samples. The extensions of Greenwood's statistic and Quesenberry and Miller's statistic in terms of overlapping k-spacings take the form

$$G_{m:n}^{(k)} = \sum_{i=1}^{m} (S_i^{(k)})^2.$$

The null hypothesis of uniformity is rejected if these statistics are too large.

- Balakrishnan et al.'s (2004) test statistic was defined as below:

$$T = \frac{\sum_{i=2}^{m-1} (m-i) G_i}{(m-2) \sum_{i=2}^{m} G_i},$$

where

$$G_i = \frac{S_i}{E(s_i)} = \frac{U_{i:m:n} - U_{i-1:m:n}}{\mu_{i:m:n} - \mu_{i-1:m:n}}.$$

In the next section, we propose a new test statistic and in Section 4, we compare it with the reviewed test statistics.

3 Proposed test

In this section, we propose a new approach for goodness-of-fit testing for normality under progressively Type-II censored data. Consider again the goodness-of-fit testing problem (1) based on $X_{1:m:n}, ..., X_{m:m:n}$, where

$$f_0(x; \mu, \sigma) = 1/\sqrt{2\pi\sigma^2} e^{-(x-\mu)^2/2\sigma^2}, \quad x \in \mathbb{R},$$

in which $\mu \in \mathbb{R}$ and $\sigma > 0$ are both unknown. Suppose $\hat{\mu}$ and $\hat{\sigma}$ are the MLEs of μ and σ based on $X_{1:m:n}, ..., X_{m:m:n}$. Because of consistency of the ML estimators, we expect $F_0(X_{i:m:n}, \hat{\mu}, \hat{\sigma})$ has the same distribution as $U_{i:m:n}$; so it is justifiable that $\frac{F_0(X_{i:m:n}, \hat{\mu}, \hat{\sigma})}{\mu_{i:m:n}} \simeq 1$. Our proposed test is based on this ratio. More precisely, define

$$\mathrm{H}_{m:n} = \frac{1}{m} \sum_{i=1}^{m} h\left(\frac{F_0(X_{i:m:n}, \hat{\mu}, \hat{\sigma})}{\mu_{i:m:n}} \right),$$

where $h : (0, \infty) \to \mathbb{R}^+$ is assumed to be continuous, decreasing on $(0, 1)$ and increasing on $(1, \infty)$ with the absolute minimum at $x = 1$ such that $h(1) = 0$. By comparison of the powers using Monte Carlo simulation for some choices h, we observed that the best choice is

$$h(x) = \frac{(x-1)^2}{x^2+1},$$

which has the maximal power. Plot of the function h is given in Figure 1.

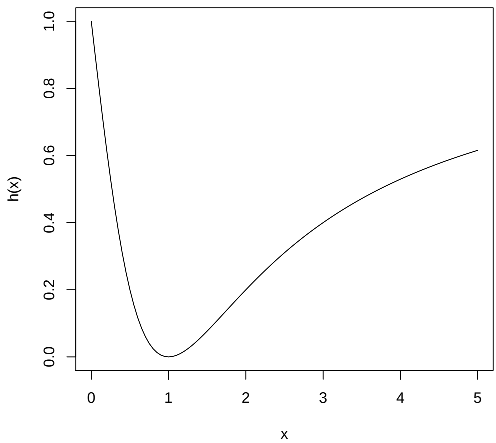

Figure 1: Plot of the function $h(x)$.

We know that MLE of μ and σ are location-scale invariant for μ and σ, respectively. Therefore under a location-scale transformation, the distribution of $\mathrm{H}_{m:n}$ does not depend on the parameters μ and σ under location-scale transformations.

It is expected that the null hypothesis of normality is rejected if the statistic $\mathrm{H}_{m:n}$ is too large. Thus, the critical region is of the form $\{\mathrm{H}_{m:n} > c\}$, for some $c > 0$. But for finding c for a size α test, the exact distribution of $\mathrm{H}_{m:n}$ can not be explicitly obtained; but the critical points can be determined by Monte Carlo simulation.

An adequate test statistic for a goodness-of-fit test problem should be consistent, i.e, with increasing sample size, it is expected that the statistic tends to a finite value, especially under H_0 tends to zero. We can not prove consistency of the proposed test statistic, but in the following it is discussed using Monte Carlo simulation. To illustrate the goal, we consider 5 various censoring schemes as the below:

- **Scheme {1}**: Progressive Type-II censoring scheme with constant removal, $\boldsymbol{r} = (1, 1, ..., 1)$, in this case $n = 2m$;

- **Scheme {2}**: a progressive Type-II censoring scheme with increasing removal, $r_i = i$, for $i = 1, 2, ..., m$, in this case $n = m(m+3)/2$;

- **Scheme {3}**: Progressive Type-II censoring scheme with decreasing removal, $r_i = m - i + 1$ for $i = 1, 2, ..., m$, thus $n = m(m+3)/2$;

- **Scheme {4}**: Type-II censoring, $r_i = 0$ for $i = 1, 2, ..., m-1$, $r_m = m/5$, hence $n = 1.2m$;
- **Scheme {5}**: Complete data, i.e., $r_i = 0$ for $i = 1, 2, ..., m$, thus $n = m$;

As it is stated, the normal model is considered as the parent model in H_0, but it can be replaced by any location-scale model due to the structure of test statistic. Against this model, we consider the following alternative models:

- Student's t distribution with ν degrees of freedom ($t_{(\nu)}$) with the density function

$$f(x;\nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)}(1+\frac{x^2}{\nu})^{-(\nu+1)/2}, \quad \nu > 0.$$

- Logistic distribution with parameters μ and σ ($L(\mu, \sigma)$) with the density function

$$f(x;\mu,\sigma) = \frac{\frac{1}{\sigma}\exp\left[\frac{-(x-\mu)}{\sigma}\right]}{\left(1-\frac{1}{\sigma}\exp\left[\frac{-(x-\mu)}{\sigma}\right]\right)^2}, \quad x \in \mathbb{R}, \mu \in \mathbb{R}, \sigma > 0.$$

- Double exponential distribution with parameters μ and σ ($DE(\mu, \sigma)$) with the density function

$$f(x;\mu,\sigma) = \frac{1}{2\sigma}\exp\left[\frac{-|x-\mu|}{\sigma}\right], \quad x \in \mathbb{R}, \mu \in \mathbb{R}, \sigma > 0.$$

For more details on these distributions, one may refer to Casella and Berger (2002).

Averages of the simulated values of statistic are reported in Table 1. In the view of this table, it is observed that under the null hypothesis of normality, the average of values of the test statistic tend to zero when m increases (and hence n increases), but under the alternative distributions such as $t_{(3)}$, $t_{(4)}$, $L(0, 1)$ and $DE(0, 1)$, the averages tend to a non zero value for all five schemes.

4 Simulation Study

In this section, we assess the power of the proposed test statistic by comparing the simulated power values with those of the test of Balakrishnan et al. (2004) and Pakyari and Balakrishnan (2013). We calculated the power of the proposed test for testing of normality against some different alternatives with simulating 10,000 random samples for some different choices of sample sizes and progressive censoring schemes. For comparative purposes, all 27 censoring schemes used by Balakrishnan et al. (2004) and Pakyari and Balakrishnan (2013) in their studies are considered again here, and these are listed in Table 2. Also the simulated critical values of $\mathrm{H}_{m:n}$ for every 27 censoring scheme has listed in Table 3. All the simulations were carried out in R software.

In Table 4, we present the estimated power of the our proposed test, Balakrishnan et al.'s (2004) T-statistic and Pakyari and Balakrishnan's (2013) test statistics when the null hypothesis stipulates normal and the alternative hypothesis corresponds to

Table 1: Averages of simulated values of $\mathrm{H}_{m:n}$ when the null hypothesis is normal distribution.

Scheme no.	n	m	$N(0,1)$	$t_{(3)}$	$t_{(4)}$	$DE(0,1)$	$L(0,1)$
{1}	50	25	0.0301	0.0617	0.0518	0.0580	0.0405
	100	50	0.0184	0.0541	0.0426	0.0470	0.0292
	200	100	0.0112	0.0501	0.0366	0.0406	0.0223
	300	150	0.0084	0.0481	0.0345	0.0384	0.0195
	400	200	0.0067	0.0475	0.0334	0.0375	0.0180
	500	250	0.0057	0.0459	0.0325	0.0368	0.0169
	600	300	0.0050	0.0397	0.0314	0.0364	0.0134
{2}	65	10	0.0639	0.0951	0.0865	0.0789	0.07356
	230	20	0.0391	0.0818	0.0695	0.0535	0.0504
	430	40	0.0241	0.0737	0.0594	0.0365	0.0346
	1890	60	0.0178	0.0682	0.0546	0.0293	0.0280
	3320	80	0.0144	0.0630	0.0525	0.0246	0.0244
	5150	100	0.0123	0.0471	0.0464	0.0224	0.0217
{3}	65	10	0.0665	0.1124	0.0994	0.0949	0.0812
	230	20	0.0449	0.1128	0.0924	0.0785	0.0636
	430	40	0.0295	0.1191	0.0922	0.0647	0.0522
	1890	60	0.0230	0.1251	0.0957	0.0579	0.0452
	3320	80	0.0188	0.1249	0.0974	0.0540	0.0425
	5150	100	0.0160	0.1195	0.0978	0.0503	0.0408
{4}	60	50	0.0170	0.0429	0.0343	0.0397	0.0247
	120	100	0.0104	0.0391	0.0291	0.0343	0.0185
	180	150	0.0077	0.0379	0.0269	0.0323	0.0158
	240	200	0.0064	0.0369	0.0258	0.0318	0.0145
	300	250	0.0051	0.0361	0.0247	0.0312	0.0135
	360	300	0.0046	0.0350	0.0243	0.0305	0.0130
	420	350	0.0041	0.0313	0.0239	0.0300	0.0101
	480	400	0.0037	0.0215	0.0193	0.0274	0.0097
{5}	50	50	0.0165	0.0371	0.0297	0.0467	0.0215
	100	100	0.0103	0.0351	0.0250	0.0265	0.0159
	200	200	0.0063	0.0334	0.0225	0.0235	0.0121
	400	400	0.0037	0.0335	0.0206	0.0218	0.0095
	800	800	0.0022	0.0334	0.0196	0.0208	0.0083
	1600	1600	0.0013	0.0334	0.0189	0.0203	0.0075
	3200	3200	0.0007	0.0334	0.0189	0.0203	0.0075

Student's t with three and four degrees of freedom, Logistic distribution and double exponential distribution. From this table, it is observed that for a symmetric heavy-tailed alternative in the case of normality test, the test statistic $\mathrm{H}_{m:n}$ has possessed

Table 2: Progressive censoring schemes used in the Monte Carlo simulations.

Scheme no.	n	m	$\boldsymbol{r} = (r_1, r_2, ..., r_m)$
[1]	20	8	$r_1 = 12, r_i = 0$ for $i \neq 1$
[2]	20	8	$r_8 = 12, r_i = 0$ for $i \neq 8$
[3]	20	8	$r_1 = r_8 = 6, r_i = 0$ for $i \neq 1, 8$
[4]	20	12	$r_1 = 8, r_i = 0$ for $i \neq 8$
[5]	20	12	$r_{12} = 8, r_i = 0$ for $i \neq 12$
[6]	20	12	$r_3 = r_5 = r_7 = r_9 = 2, r_i = 0$ for $i \neq 3, 5, 7, 9$
[7]	20	16	$r_1 = 4, r_i = 0$ for $i \neq 1$
[8]	20	16	$r_{16} = 4, r_i = 0$ for $i \neq 16$
[9]	20	16	$r_5 = 4, r_i = 0$ for $i \neq 5$
[10]	40	10	$r_1 = 30, r_i = 0$ for $i \neq 1$
[11]	40	10	$r_{10} = 30, r_i = 0$ for $i \neq 10$
[12]	40	10	$r_1 = r_5 = r_{10} = 10, r_i = 0$ for $i \neq 1, 5, 10$
[13]	40	20	$r_1 = 20, r_i = 0$ for $i \neq 1$
[14]	40	20	$r_{20} = 20, r_i = 0$ for $i \neq 20$
[15]	40	20	$r_i = 1$, for $r_i = 1, 2, ..., 20$
[16]	40	30	$r_1 = 10, r_i = 0$ for $i \neq 1$
[17]	40	30	$r_{30} = 10, r_i = 0$ for $i \neq 30$
[18]	40	30	$r_1 = r_{30} = 5, r_i = 0$ for $i \neq 1, 30$
[19]	60	20	$r_1 = 40, r_i = 0$ for $i \neq 1$
[20]	60	20	$r_{20} = 40, r_i = 0 for i \neq 20$
[21]	60	20	$r_1 = r_{20} = 10, r_{10} = 20 r_i = 0$ for $i \neq 1, 10, 20$
[22]	60	40	$r_1 = 20, r_i = 0$ for $i \neq 1$
[23]	60	40	$r_{40} = 20, r_i = 0$ for $i \neq 40$
[24]	60	40	$r_{2i-1} = 1, r_{2i} = 0,$ for $i = 1, 2, ..., 20$
[25]	60	50	$r_1 = 10, r_i = 0$ for $i \neq 1$
[26]	60	50	$r_{50} = 10, r_i = 0$ for $i \neq 1$
[27]	60	50	$r_1 = r_{50} = 5, r_i = 0$ for $i \neq 1, 50$

Table 3: Simulated critical values of $\mathrm{H}_{m:n}$.

Scheme no.	$\mathrm{H}_{m:n}$	Scheme no.	$\mathrm{H}_{m:n}$	Scheme no.	$\mathrm{H}_{m:n}$
[1]	0.1069	[10]	0.1033	[19]	0.0633
[2]	0.1062	[11]	0.0941	[20]	0.0595
[3]	0.1060	[12]	0.0971	[21]	0.0621
[4]	0.0802	[13]	0.0588	[22]	0.0351
[5]	0.0793	[14]	0.0573	[23]	0.0358
[6]	0.0846	[15]	0.0602	[24]	0.0370
[7]	0.0646	[16]	0.0424	[25]	0.0296
[8]	0.0661	[17]	0.0431	[26]	0.0300
[9]	0.0671	[18]	0.0425	[27]	0.0298

better power than Balakrishnan et al.'s (2004) T-statistic and Pakyari and Balakrishnan's (2013) test statistics in 78 out of 108 situations. Also, when $n = 20$ in the Student's t distribution with three degrees of freedom in 4 out of 9 situations, in the Student's t distribution with four degrees of freedom in 5 out of 9, in the Logistic $(0, 1)$ in 6 out of 9 situations and in the double exponential $(0, 1)$ in 7 out of 9 situations, in the case $n = 40$ in the Student's t distribution with three degrees of freedom in 7 out of 9 situations, in the Student's t distribution with four degrees of freedom in 7 out of 9, in the Logistic $(0, 1)$ in 7 out of 9 situations and in the double exponential $(0, 1)$ in 7 out of 9 situations and in the cases $n = 60$ in the Student's t distribution with three degrees of freedom in 6 out of 9 situations, in the Student's t distribution with four degrees of freedom in 8 out of 9, in the Logistic $(0, 1)$ in 8 out of 9 situations and in the double exponential $(0, 1)$ in 6 out of 9 situations our test statistic possessed better power than Balakrishnan et al.'s (2004) T-statistic and Pakyari and Balakrishnan's (2013) test statistics. Also, in early censoring schemes ([1],[4],[7],[10],[13],[16],[19],[22],[25]), the $G^{(2)}_{m:n}$ statistic has the most power in 6 out 36 situations, the $G^{(3)}_{m:n}$ statistic has the most power in 11 out 36 situations, the $Q_{m:n}$ statistic has the most power in 1 out 36 situations, the $T^{(2)}_{m:n}$ statistic has the most power in 4 out 36 situations, the T-statistic has the most power in 4 out 36 situations and the $\mathrm{H}_{m:n}$ statistic has the most power in 10 out 36 situations. In addition, in non-early censoring schemes ([2],[3],[5],[6],[8],[9],[11],[12],[14],[15],[17],[18],[20],[21],[23],[24],[26],[27]), the $G^{(3)}_{m:n}$ statistic has the most power in 1 out 72 situations, the $Q_{m:n}$ statistic has the most power in 1 out 72 situations, the T-statistic has the most power in 2 out 72 situations and the $\mathrm{H}_{m:n}$ statistic has the most power in 68 out 72 situations. Also note that, as one would normally expect, it can be observed from the values in the Table 4 that the power increases as the degree of censoring $(1 - m/n)$ decreases. Finally, based on this results and comparative findings, we recommend the use of $G^{(3)}_{m:n}$ and $\mathrm{H}_{m:n}$ statistics for the case of early censoring and the use of $\mathrm{H}_{m:n}$ statistic for the case of non-early censoring.

Table 4: Estimated powers of the statistics with $\alpha = 0.1$.

Scheme no.	dist.	$C^+_{m:n}$	$C^-_{m:n}$	$C_{m:n}$	$K_{m:n}$	$T^{(1)}_{m:n}$	$T^{(2)}_{m:n}$	$G_{m:n}$	$Q_{m:n}$	$G^{(2)}_{m:n}$	$G^{(3)}_{m:n}$	T	$\mathrm{H}_{m:n}$
[1]	$t_{(3)}$	0.2224	0.1112	0.2251	0.1681	0.2266	0.2158	0.2438	0.2528	**0.2676**	0.2341	0.2567	0.2638
	$t_{(4)}$	0.1807	0.1016	0.1770	0.1303	0.1738	0.1661	0.1946	0.2043	**0.2211**	0.1920	0.2056	0.2172
	$L(0,1)$	0.1232	0.1003	0.1114	0.0944	0.1199	0.1122	0.1369	0.1376	**0.1509**	0.1305	0.1217	0.1408
	$DE(0,1)$	0.1912	0.0943	0.1803	0.1251	0.1725	0.1668	0.2188	0.2322	**0.2377**	0.2216	0.1909	0.2286
[2]	$t_{(3)}$	0.2450	0.0507	0.1463	0.1459	0.1541	0.1554	0.2057	0.2272	0.1579	0.0754	0.2461	**0.2895**
	$t_{(4)}$	0.1988	0.0616	0.1221	0.1234	0.1254	0.1257	0.1678	0.1839	0.1335	0.0766	0.1965	**0.2380**
	$L(0,1)$	0.1440	0.0682	0.0937	0.0985	0.1069	0.0989	0.1330	0.1321	0.1039	0.0956	0.1330	**0.1585**
	$DE(0,1)$	0.2222	0.0489	0.1198	0.1313	0.1310	0.1317	0.1827	0.2078	0.1236	0.0694	0.1746	**0.2555**
[3]	$t_{(3)}$	0.2592	0.0471	0.1887	0.1288	0.1802	0.1771	0.2316	0.2541	0.2883	0.2570	0.2543	**0.3000**
	$t_{(4)}$	0.2066	0.0572	0.1480	0.1098	0.1433	0.1385	0.1871	0.2004	0.2276	0.2085	0.2018	**0.2382**
	$L(0,1)$	0.1458	0.0715	0.1094	0.0920	0.1157	0.1092	0.1279	0.1345	0.1577	0.1364	0.1297	**0.1654**
	$DE(0,1)$	0.2438	0.0402	0.1671	0.1210	0.1774	0.1673	0.2116	0.2423	0.2569	0.2372	0.1833	**0.2902**
[4]	$t_{(3)}$	0.2233	0.1464	0.2510	0.2000	0.2598	0.2583	0.2523	0.2716	0.3036	0.2802	**0.3071**	0.3008
	$t_{(4)}$	0.1772	0.1200	0.1879	0.1505	0.1921	0.1916	0.2021	0.2161	0.2424	0.2234	**0.2490**	0.2364
	$L(0,1)$	0.1235	0.0972	0.1318	0.1017	0.1180	0.1199	0.1416	0.1471	**0.1748**	0.1669	0.1411	0.1673
	$DE(0,1)$	0.1915	0.1195	0.2149	0.1647	0.2148	0.2029	0.2332	0.2661	**0.2904**	0.2734	0.2340	0.2702
[5]	$t_{(3)}$	0.2630	0.0448	0.1848	0.1526	0.1892	0.1873	0.2579	0.2807	0.3003	0.2657	0.3190	**0.3526**
	$t_{(4)}$	0.2036	0.0546	0.1442	0.1229	0.1462	0.1438	0.2038	0.2227	0.2335	0.2112	0.2484	**0.2797**
	$L(0,1)$	0.1463	0.0721	0.1083	0.0929	0.1066	0.1136	0.1393	0.1591	0.1585	0.1380	0.1311	**0.1927**
	$DE(0,1)$	0.2783	0.0343	0.1835	0.1485	0.1819	0.1890	0.2483	0.3087	0.3112	0.2776	0.1980	**0.3682**
[6]	$t_{(3)}$	0.0952	0.2467	0.1784	0.1908	0.2123	0.2259	0.2703	0.2895	0.2732	0.2211	**0.3386**	0.3367
	$t_{(4)}$	0.0819	0.1906	0.1373	0.1491	0.1626	0.1693	0.2159	0.2277	0.2173	0.1803	0.2727	**0.2748**
	$L(0,1)$	0.0767	0.1309	0.1092	0.1013	0.1064	0.1076	0.1497	0.1541	0.1435	0.1391	0.1434	**0.1826**
	$DE(0,1)$	0.0843	0.2266	0.1560	0.1698	0.1807	0.1941	0.2406	0.2847	0.2605	0.2406	0.2051	**0.3234**
[7]	$t_{(3)}$	0.2183	0.1881	0.2718	0.2332	0.2894	0.2865	0.2694	0.2832	0.2942	0.2586	**0.3407**	0.3193
	$t_{(4)}$	0.1698	0.1433	0.2018	0.1629	0.2102	0.2080	0.2119	0.2216	0.2364	0.2076	**0.2755**	0.2645
	$L(0,1)$	0.1205	0.1086	0.1183	0.0984	0.1231	0.1249	0.1484	0.1557	0.1540	0.1518	0.1539	**0.1707**
	$DE(0,1)$	0.1926	0.1646	0.2253	0.1824	0.2353	0.2485	0.2507	0.2738	0.2740	0.2617	0.2668	**0.2850**
[8]	$t_{(3)}$	0.2533	0.0740	0.2055	0.1652	0.2048	0.2021	0.2799	0.3089	0.3164	0.2917	0.3309	**0.3684**
	$t_{(4)}$	0.1963	0.0701	0.1581	0.1251	0.1564	0.1558	0.2216	0.2406	0.2464	0.2266	0.2607	**0.2987**
	$L(0,1)$	0.1374	0.0761	0.1137	0.0954	0.1071	0.1090	0.1505	0.1650	0.1668	0.1400	0.1391	**0.1900**
	$DE(0,1)$	0.2665	0.0625	0.2117	0.1632	0.2106	0.2097	0.2842	0.3321	0.3240	0.3061	0.2100	**0.3941**
[9]	$t_{(3)}$	0.1531	0.2330	0.2292	0.2185	0.2621	0.2663	0.2839	0.2997	0.2805	0.2255	**0.3479**	0.3204
	$t_{(4)}$	0.1178	0.1722	0.1679	0.1584	0.1911	0.2226	0.2381	0.2216	0.1805	**0.2810**	0.2607	0.2681
	$L(0,1)$	0.0944	0.1156	0.1169	0.1058	0.1124	0.1225	0.1594	**0.1632**	0.1523	0.1300	0.1447	0.1620
	$DE(0,1)$	0.1326	0.1940	0.2005	0.1843	0.2153	0.2279	0.2756	0.2964	0.2940	0.2189	0.2237	**0.2992**
[10]	$t_{(3)}$	0.3111	0.1037	0.3002	0.2300	0.3094	0.2918	0.3258	0.3457	0.3408	**0.3507**	0.3098	0.3380
	$t_{(4)}$	0.2386	0.0880	0.2204	0.1637	0.2253	0.2111	0.2521	0.2640	0.2750	**0.2759**	0.2432	0.2577
	$L(0,1)$	0.1514	0.0746	0.1295	0.1096	0.1328	0.1269	0.1585	0.1581	0.1691	**0.1960**	0.1477	0.1584
	$DE(0,1)$	0.2549	0.0782	0.2242	0.1646	0.2329	0.2091	0.2691	0.2883	0.2814	**0.3279**	0.2313	0.2556
[11]	$t_{(3)}$	0.3057	0.0404	0.1832	0.1939	0.1881	0.1952	0.2153	0.2417	0.0773	0.0473	0.03170	**0.3527**
	$t_{(4)}$	0.2384	0.0488	0.1411	0.1539	0.1462	0.1520	0.1728	0.1920	0.0753	0.0589	0.2646	**0.2847**
	$L(0,1)$	0.1541	0.0704	0.1024	0.1173	0.0967	0.1101	0.1180	0.1301	0.0745	0.0855	0.1312	**0.1800**
	$DE(0,1)$	0.2041	0.0484	0.1156	0.14434	0.1178	0.1268	0.1489	0.1594	0.0640	0.0540	0.1545	**0.2384**

Table 4: Continued

Scheme no.	dist.	$C^+_{m:n}$	$C^-_{m:n}$	$C_{m:n}$	$K_{m:n}$	$T^{(1)}_{m:n}$	$T^{(2)}_{m:n}$	$G_{m:n}$	$Q_{m:n}$	$G^{(2)}_{m:n}$	$G^{(3)}_{m:n}$	T	$\mathrm{H}_{m:n}$
[12]	$t_{(3)}$	0.1814	0.0720	0.1291	0.1465	0.1715	0.1835	0.2912	0.3369	0.3318	0.2963	0.3703	**0.4090**
	$t_{(4)}$	0.1413	0.0594	0.1048	0.1082	0.1263	0.1393	0.2261	0.2566	0.2577	0.2303	0.2819	**0.3171**
	$L(0,1)$	0.1091	0.0614	0.0841	0.0842	0.897	0.0875	0.1341	0.1686	0.1528	0.1483	0.1453	**0.2013**
	$DE(0,1)$	0.1527	0.0372	0.0987	0.0914	0.1201	0.1205	0.2096	0.2754	0.2429	0.2413	0.2469	**0.3116**
[13]	$t_{(3)}$	0.1363	0.2004	0.3627	0.3282	0.3897	0.3860	0.3567	0.3866	0.4212	**0.4415**	0.3797	0.4318
	$t_{(4)}$	0.2544	0.1407	0.2617	0.2208	0.2738	0.2725	0.2689	0.2940	0.3319	**0.3501**	0.3018	0.3369
	$L(0,1)$	0.1415	0.1015	0.1456	0.1200	0.1407	0.1471	0.1645	0.1700	0.1828	**0.2236**	0.1651	0.2040
	$DE(0,1)$	0.2980	0.1557	0.2974	0.2668	0.3148	0.3241	0.2969	0.33276	0.3497	**0.3984**	0.2635	0.3751
[14]	$t_{(3)}$	0.4075	0.0375	0.2934	0.2543	0.3087	0.3124	0.3095	0.3592	0.3960	0.3913	0.4820	**0.5050**
	$t_{(4)}$	0.3070	0.0395	0.2076	0.1807	0.2100	0.2145	0.2359	0.2706	0.3071	0.3049	0.3729	**0.3886**
	$L(0,1)$	0.1848	0.0623	0.1267	0.1070	0.1177	0.1229	0.1504	0.1612	0.1758	0.1968	0.1374	**0.2325**
	$DE(0,1)$	0.4194	0.0186	0.2619	0.2113	0.2737	0.2944	0.2724	0.3205	0.3443	0.3652	0.2506	**0.4484**
[15]	$t_{(3)}$	0.1300	0.3242	0.2457	0.2549	0.2921	0.3088	0.3628	0.4177	0.4143	0.4054	0.4822	**0.5062**
	$t_{(4)}$	0.0962	0.2350	0.1700	0.1765	0.2003	0.2096	0.2773	0.3184	0.3150	0.3109	0.3797	**0.3987**
	$L(0,1)$	0.0751	0.1377	0.1073	0.1152	0.1136	0.1211	0.1723	0.1905	0.1932	0.1932	0.1624	**0.2454**
	$DE(0,1)$	0.1071	0.2996	0.2098	0.2307	0.2793	0.2967	0.3308	0.4087	0.3884	0.4057	0.2597	**0.4997**
[16]	$t_{(3)}$	0.3546	0.2936	0.4107	0.3990	0.4461	0.4501	0.3786	0.4153	0.4316	0.4186	0.4152	**0.4737**
	$t_{(4)}$	0.2499	0.2000	0.2784	0.2635	0.3096	0.3087	0.2802	0.3084	0.3285	0.3197	0.3281	**0.3666**
	$L(0,1)$	0.1411	0.1157	0.1412	0.1272	0.1521	0.1467	0.1626	0.1852	0.1987	0.2013	0.1903	**0.2168**
	$DE(0,1)$	0.3175	0.2432	0.3443	0.3455	0.3900	0.3992	0.3292	0.3717	0.3921	0.4011	0.2906	**0.4573**
[17]	$t_{(3)}$	0.3961	0.0941	0.3201	0.2794	0.3346	0.3410	0.3847	0.4430	0.4615	0.4671	0.4998	**0.5470**
	$t_{(4)}$	0.2920	0.0661	0.2259	0.1878	0.2373	0.2387	0.2865	0.3333	0.3481	0.3508	0.3894	**0.4317**
	$L(0,1)$	0.1695	0.0586	0.1278	0.1057	0.1246	0.1386	0.1648	0.1950	0.2115	0.2326	0.1632	**0.2567**
	$DE(0,1)$	0.4396	0.0527	0.3429	0.2903	0.3656	0.3753	0.3772	0.4531	0.4757	0.5266	0.2870	**0.5688**
[18]	$t_{(3)}$	0.3741	0.1213	0.3201	0.2835	0.3379	0.3416	0.3863	0.4400	0.4490	0.4538	0.4439	**0.5330**
	$t_{(4)}$	0.2769	0.0849	0.2244	0.1925	0.2365	0.2402	0.2889	0.3318	0.3426	0.3452	0.3461	**0.4115**
	$L(0,1)$	0.1582	0.0726	0.1246	0.1061	0.1395	0.1321	0.1864	0.1985	0.2037	0.2036	0.1576	**0.2479**
	$DE(0,1)$	0.3883	0.0976	0.3167	0.2919	0.3603	0.3329	0.3677	0.4269	0.4321	0.4699	0.2503	**0.5099**
[19]	$t_{(3)}$	0.4230	0.1869	0.4158	0.3865	0.4521	0.4469	0.4163	0.4513	0.4404	**0.5027**	0.3904	0.4785
	$t_{(4)}$	0.3098	0.1264	0.2962	0.2576	0.3154	0.3087	0.3096	0.3428	0.3365	**0.3904**	0.3073	0.3710
	$L(0,1)$	0.1777	0.0845	0.1550	0.1286	0.1536	0.1423	0.1826	0.1988	0.2162	**0.2221**	0.1680	0.2102
	$DE(0,1)$	0.3499	0.1380	0.3285	0.2934	0.3425	0.3261	0.3390	**0.3982**	0.3413	0.3860	0.2666	0.3971
[20]	$t_{(3)}$	0.4547	0.0220	0.3208	0.2860	0.3392	0.3472	0.2853	0.3377	0.2925	0.2383	0.5208	**0.5308**
	$t_{(4)}$	0.3478	0.0312	0.2229	0.2003	0.2332	0.2406	0.2152	0.2517	0.2161	0.1713	0.4021	**0.4154**
	$L(0,1)$	0.1985	0.0512	0.1223	0.1136	0.1235	0.1286	0.1461	0.1599	0.1195	0.1085	0.1399	**0.2434**
	$DE(0,1)$	0.3416	0.0224	0.2026	0.1761	0.2149	0.2176	0.2058	0.2468	0.1533	0.1207	0.2026	**0.3815**
[21]	$t_{(3)}$	0.1537	0.2744	0.2065	0.2770	0.2871	0.3231	0.3994	0.4680	0.4739	0.4303	0.5826	**0.6001**
	$t_{(4)}$	0.1051	0.1762	0.1354	0.1757	0.1839	0.2116	0.2988	0.3540	0.3527	0.3163	0.4496	**0.4700**
	$L(0,1)$	0.0830	0.0999	0.0826	0.0891	0.1023	0.1047	0.1772	0.2033	0.1978	0.1796	0.1595	**0.2718**
	$DE(0,1)$	0.1355	0.1732	0.1407	0.1974	0.2154	0.2346	0.3432	0.4297	0.4373	0.4069	0.2931	**0.5277**
[22]	$t_{(3)}$	0.4571	0.3707	0.5185	0.5201	0.5653	**0.5719**	0.4529	0.5039	0.5243	0.5370	0.4446	0.5715
	$t_{(4)}$	0.1963	0.0701	0.1581	0.1251	0.1564	0.1558	0.2216	0.2406	0.2464	0.2266	0.2607	**0.4462**
	$L(0,1)$	0.1503	0.1237	0.1621	0.1491	0.1786	0.1764	0.1805	0.2000	0.2090	0.2298	0.1911	**0.2441**
	$DE(0,1)$	0.3981	0.2911	0.4360	0.4678	0.5049	**0.5090**	0.3715	0.4322	0.4402	0.4912	0.3024	0.4984

Table 4: Continued

Scheme no.	Dis.	$C^{+}_{m:n}$	$C^{-}_{m:n}$	$C_{m:n}$	$K_{m:n}$	$T^{(1)}_{m:n}$	$T^{(2)}_{m:n}$	$G_{m:n}$	$Q_{m:n}$	$G^{(2)}_{m:n}$	$G^{(3)}_{m:n}$	T	$\mathrm{H}_{m:n}$
[23]	$t_{(3)}$	0.5234	0.1172	0.4238	0.3831	0.4550	0.4567	0.4385	0.5148	0.5779	0.5981	0.6351	**0.6714**
	$t_{(4)}$	0.3841	0.0652	0.2840	0.2454	0.3064	0.3083	0.3228	0.3847	0.4409	0.4929	0.2607	**0.5298**
	$L(0,1)$	0.2163	0.0518	0.1472	0.1265	0.1473	0.1502	0.1886	0.2124	0.2422	0.2622	0.1712	**0.3148**
	$DE(0,1)$	0.6119	0.0461	0.4696	0.4136	0.5146	0.5225	0.4309	0.5162	0.6080	0.6556	0.4060	**0.6967**
[24]	$t_{(3)}$	0.3141	0.4451	0.4329	0.4479	0.4833	0.4927	0.4523	0.5161	0.5161	0.5193	0.5263	**0.6261**
	$t_{(4)}$	0.1950	0.3175	0.2862	0.2975	0.3233	0.3324	0.3371	0.3890	0.3876	0.3912	0.4167	**0.4838**
	$L(0,1)$	0.0990	0.1656	0.1488	0.1408	0.1557	0.1548	0.1984	0.2225	0.2316	0.2301	0.1749	**0.2938**
	$DE(0,1)$	0.2946	0.4103	0.4173	0.4244	0.4865	0.4846	0.4079	0.4856	0.4971	0.5211	0.3063	**0.5832**
[25]	$t_{(3)}$	0.4838	0.4407	0.5496	0.5684	0.6065	**0.6245**	0.4657	0.5189	0.5224	0.5280	0.4634	0.5940
	$t_{(4)}$	0.3296	0.2879	0.3724	0.3788	0.4189	0.4261	0.3361	0.3809	0.3902	0.3910	0.36369	**0.4659**
	$L(0,1)$	0.1552	0.1360	0.1606	0.1605	0.1834	0.1788	0.1849	0.1992	0.2157	0.2224	0.1977	**0.2639**
	$DE(0,1)$	0.4304	0.3850	0.4865	0.5501	0.5685	**0.5850**	0.3942	0.4436	0.4715	0.4904	0.3039	0.5128
[26]	$t_{(3)}$	0.4908	0.1752	0.4140	0.4005	0.4492	0.4552	0.4645	0.5355	0.5497	0.5744	0.5754	**0.6567**
	$t_{(4)}$	0.3548	0.1029	0.2788	0.2605	0.3021	0.3077	0.3456	0.3989	0.4124	0.4333	0.4447	**0.5107**
	$L(0,1)$	0.1839	0.0697	0.1562	0.1206	0.1558	0.1527	0.1997	0.2219	0.2333	0.2504	0.1666	**0.3074**
	$DE(0,1)$	0.5219	0.1568	0.4641	0.4382	0.5160	0.4952	0.4495	0.5341	0.5499	0.6007	0.3264	**0.6580**
[27]	$t_{(3)}$	0.4670	0.2110	0.4169	0.4109	0.4574	0.4616	0.4619	0.5236	0.5275	0.5526	0.4998	**0.6217**
	$t_{(4)}$	0.3330	0.1286	0.2819	0.2719	0.3149	0.3146	0.3428	0.3917	0.3980	0.4155	0.3871	**0.4983**
	$L(0,1)$	0.1796	0.0861	0.1443	0.1315	0.1537	0.1545	0.1905	0.2172	0.2170	0.2393	0.1635	0.**2842**
	$DE(0,1)$	0.4800	0.2140	0.4486	0.4521	0.5073	0.5037	0.4226	0.4995	0.5100	0.5325	0.2744	**0.6037**

5 Illustrative data analyses

In this section, the wire connection strength data from Nelson (1982), (Table 5.1, p. 111) are considered. These data, first studied by King (1971), concern the breaking strength of 23 wire connections. The wires were bonded at one end to a semiconductor wafer and at the other end to a terminal post. The first two and the last one of the observations were eliminated from the analysis due to validity of the data; see Nelson (1982), for more details. Pakyari and Balakrishnan (2013) randomly generated a progressively Type-II censored sample of size $m = 10$ from $n = 20$ observations. Table 5 presents the data and the corresponding progressive censoring scheme. The possibility of fitting a normal model to the data was done by Nelson (1982), and we also tested for normality. Table 6 presents the test statistics and their corresponding p-values. The normal model is strongly supported by all the test statistics for describing the wire connection strength data. Results in Table 6 show that the test statistic, $\mathrm{H}_{m:n}$, agrees with the other statistics.

Table 5: Wire connection strength data and the progressive Type-II censoring scheme.

i	1	2	3	4	5	6	7	8	9	10
$x_{i:m:n}$	550	750	950	1150	1150	1150	1350	1450	1550	1850
r_i	0	2	1	0	3	0	0	2	0	2

Table 6: Test statistics and the corresponding p-values for the data given in Table 5 for testing the normal distribution.

Criterion	$C^+_{m:n}$	$C^-_{m:n}$	$C_{m:n}$	$K_{m:n}$	$T^{(1)}_{m:n}$	$T^{(2)}_{m:n}$
Test statistic	0.0946	0.0893	0.0946	0.1839	0.0021	0.0352
p-value	0.6576	0.3809	0.7057	0.5364	0.8020	0.8735
Criterion	$G_{m:n}$	$Q_{m:n}$	$G^{(2)}_{m:n}$	$G^{(3)}_{m:n}$	T	$\mathrm{H}_{m:n}$
Test statistic	6.8499	10.9208	26.7465	63.8562	0.4568	0.3220
p-value	0.7152	0.6476	0.6879	0.6689	0.6450	0.8091

Conclusions

In this paper, we proposed a simple and powerful test for normality based on progressively Type-II censored order statistics and compared this new test with all previously proposed normality tests. Using a simulation study, consistency of our test was illustrated and also power of the test for some various alternatives were obtained and reported. It was apparent from Table 4 that none of the tests considered performs better than all other tests against all alternatives. Comparing with other tests, however, the proposed test $\mathrm{H}_{m:n}$, was the most powerful with respect to approximately all censoring schemes. Then, the performance of our test was examined for a real data set and the results were completely coincided with the other tests.

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References

Balakrishnan, N. (2007). Progressive censoring methodology: an appraisal (with discussions). *Test*, **16**, 211–296.

Balakrishnan, N. and Aggarwala, R. (2000). *Progressive Censoring: Theory, Methods, and Applications.* Birkhäuser, Boston.

Balakrishnan, N., Habibi Rad, A. and Arghami, N.R. (2007). Testing exponentiality based on Kullback–Leibler information with progressively Type-II censored data. *IEEE Transactions on Reliability*, **56**, 301–307.

Balakrishnan, N., Ng, H.K.T. and Kannan, N. (2002). A test of exponentiality based on spacings for progressively Type-II censored data, in Huber-Carol, C., Balakrishnan, N., Nikulin, M.S. and Mesbah, M., eds., *Goodness-of-Fit Tests and Model Validity*, Birkhäuser, Boston.

Balakrishnan, N., Ng, H.K.T. and Kannan N. (2004). Goodness-of-fit tests based on spacings for progressively Type-II censored data from a general location-scale distribution. *IEEE Transaction on Reliability*, **53**, 349–356.

Casella, G. and Berger, R.L. (2002). *Statistical Inference.* 2nd Edition, Duxbury Press.

D'Agostino, R.B. and Stephens, M.A. (eds.) (1986). Goodness-of-Fit Techniques. Marcel Dekker, NewYork.

Greenwood, M. (1964). The statistical study of infectious diseases. *Journal of the Royal Statistical Society Series A*, **109**, 85–110.

Hartley, H.O. and Pfaffenberger, R.C. (1972). Quadratic forms in order statistics used as goodness-of-fit criteria. *Biometrika*, **59**, 605–611.

Huber-Carol, C., Balakrishnan, N., Nikulin, M.S. and Mesbah, M.,(eds.) (2002). *Goodness-of-Fit Tests and Model Validity.* Birkhäuser, Boston, MA.

King, J.R. (1971). *Probability Charts for Decision Making.* Industrial Press, New York.

Moran, P.A.P. (1951). The random division of an interval–Part I. *Journal of the Royal Statistical Society, Series B*, **9**, 92–98.

Nadeb, H., and Torabi, H. (2018). A goodness of fit test for location-scale family of distributions under Type-II progressive censoring. *Journal of Statistical Sciences*, (Accepted).

Nelson, W. (1982). *Applied Life Data Analysis.* John Wiley & Sons, New York.

Pakyari, R. and Balakrishnan, N. (2012). A general purpose approximate goodness-of-fit for progressively Type-II censored data. *IEEE Transactions on Reliability*, **61**(1), 238–244.

Pakyari, R. and Balakrishnan, N. (2013). Goodness-of-fit tests for progressively Type-II censored data from location–scale distributions. *Journal of Statistical Computation and Simulation*, **83** (1), 167–178.

Quesenberry, C.P. and Miller, F.L. Jr (1975). Power studies of some tests for uniformity. *Journal of Statistical Computation and Simulation*, **5**, 169–191.

Torabi, H (2006). A new method for hypotheses testing using spacing. *Statistics and Probability Letters*, **76**, 1345–1347.

Torabi, H. (2008). A general method for estimating hypotheses testing using spacing. *Journal of Statistical Theory and Applications*, **8(2)**, 163–168.

Wang, B. (2008). Goodness-of-fit test for the exponential distribution based on progressively Type-II censored sample, *Journal of Statistical Computation and Simulation*, **78**, 125–132.