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Computational approach test in one-way fixed effects ANOVA models of log-normal samples

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Abstract: We apply a recently developed computational approach test (CAT) to the one-way fixed effects ANOVA models of log-normal data with unequal variances. The merits of the proposed test are numerically compared with the existing tests - the James second order test, the Welch test and the Alexander-Govern test - with respect to their sizes and powers in different combinations of parameters and various sample sizes. The simulation results demonstrate that the proposed method is satisfactory: its type I error probability is very close to the nominal level. We illustrate these approaches using a real example.

Keywords: Log-normal Distribution, Computational Approach Test, Power; Actual size.
Mathematics Subject Classification (2010): 62F03, 62F40.

1 Introduction

The skewed distributions are particularly common when mean values are small, variances are large and values cannot be negative (for example lengths of latent periods of infectious diseases), and often closely fit the log-normal distribution. The log-normal distribution has been widely used in medical, biological and economic studies, where data are positive and have a right-skewed distribution. Various other motivations and applications of the log-normal distribution can be found in Johnson et al. (1994) and Crow and Shimizu (1987). Let X_{ij}, $i = 1, \cdots, j = 1, \cdots, n_i$, k independent random samples from log-normal distributions, i.e.

$$Y_{ij} = \log(X_{ij}) \sim N(\mu_i, \sigma_i^2)$$

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Also, let $\varphi_i = E(X_{ij}) = exp(\mu_i + \frac{1}{2}\sigma_i^2)$ denotes the mean of the i-th population. Suppose that we are interested in testing

$$H_0 : \varphi_1 = \varphi_2 = \cdots = \varphi_k \qquad vs. \qquad H_A : not\ all\ the \quad \varphi_i \quad are\ equal. \tag{1}$$

Then the testing problem (1) is equivalent to testing

$$H_0 : \eta_i = \eta \qquad vs. \qquad H_A : not\ all\ the \quad \eta_i \quad are\ equal. \tag{2}$$

where $\eta_i = \log(\varphi_i) = \mu_i + \frac{1}{2}\sigma_i^2$, and η is unspecified.
It is well-known that the maximum likelihood estimators (MLEs) for μ_i and σ_i^2 are $\bar{Y}_i$ and S_i^2, respectively, where

$$\bar{Y} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij} \qquad and \qquad S_i^2 = \frac{1}{n_i}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}^2). \tag{3}$$

Therefore, the MLE of η_i is given by $\hat{\eta}_i = \hat{\mu}_i + \frac{1}{2}\hat{\sigma}_i^2$ which is distributed approximately normal with the mean η_i and variance $\nu_i = \frac{\sigma_i^2}{n_i} + \frac{(n_i-1)\sigma_i^4}{2n_i^2}$.
Drawing inferences in the mean or the log-transformed mean on the log-normal population is an interesting topic. Guo and Luh (2000) applied the James (1951) second order test, the Welch (1951) test and the Alexander and Govern (1994) test to hypothesis testing in (1). They showed that the actual sizes of these methods are better than conventional F test for log-transformed data and F test for original data. In this paper, we first review these methods, and then we apply the CAT for testing (1). The CAT was introduced by Pal et al. (2007), generally for testing about parameters. This approach has been applied to several problems: Chang and Pal (2008) applied the CAT for the well-known BehrensFisher problem. This approach is also used for testing equality of means of several normal populations by Chang et al. (2010) when the variances are equal, and by Gökpınar and Gökpınar (2012) when the variances are unequal.
The advantage of the CAT is that, after adopting a suitable parametric model for a given dataset the researcher can draw statistical inferences through a series of computational steps and does not need to know the sufficient statistics involved, the sampling distribution of the test statistic or how to get the critical value of the test statistic.
For testing the equality of means of several lognormal populations, our simulation studies show that the CAT has a type I error probability very close to the nominal level. However, the other methods are more liberal even for large samples. Only, the actual sizes of the James, Welch and Alexander- Govern tests are close to the nominal level where σ_i^2s are close to each other.
This paper is organized as follows: In Section 2, we first review the proposed approaches by Guo and Luh (2000) for testing the equality of log-normal means. Then, we apply our method for this problem. We compare the proposed method with the other methods by some simulated data sets and a real data example in Section 3. Finally, some concluding remarks are given in Section 4.

2 The methods

Guo and Luh (2000) defined the sample t statistic for the i-th sample as

$$T_i = \sqrt{w_i}\left(\bar{Y}_i + \frac{n_i S_i^2}{2(n_i-1)} - \bar{w}\right), \qquad i = 1, ..., k,$$

where

$$w_i = \left(\frac{S_i^2}{(n_i-1)} + \frac{{n_i}^2 S_i^4}{2(n_i-1)^3}\right)^{-1} \quad , \qquad \bar{w} = \frac{1}{Q}\sum_{i=1}^{k} w_i \left(\bar{Y}_i + \frac{n_i S_i^2}{2(n_i-1)}\right),$$

and $Q = \sum_{i=1}^{k} w_i$ Using the statistic T_i, they proposed three methods for testing equality means in log-normal distributions.

2.1 Welch method

The Welch test statistic for testing (1) is

$$w = \frac{\sum_{i=1}^{k} T_i^2}{(G+1)(k-1)},$$

where

$$G = 2(k-2)(k^2-1)^{-1}\sum_{i=1}^{k} (1 - w_i/Q)^2 \Big/ (n_i - 1).$$

Therefore, H_0 is rejected if $W > F_\alpha(k-1, \tau)$ where

$$\tau = \frac{k^2-1}{3}\{\sum_{i=1}^{k} (1 - w_i/Q)^2 \Big/ (n_i-1)\}^{-1},$$

and $F_\alpha(k-1, \tau)$ is $100(1-\alpha)$ percentile of the F distribution with k-1 and τ degrees of freedom.

2.2 Alexander- Govern method

The Alexander- Govern test statistic for testing (1) is

$$AG = \sum_{i=1}^{k} Z_i^2,$$

where $Z = C_i + (C_i^3 + 3C_i)\big/a_i - D_i/E_i$

$$A_i = n_i - 1.5, \qquad a_i = 48A_i^2, \qquad C_i = \sqrt{A_i \log(1 + T_i^2/(n_i-1))},$$

$$D_i = 4C_i^7 + 33C_i^2 + 855C_i, \qquad E_i = 10a_i^2 + 8a_i C_i^4 + 1000a_i.$$

Therefore, H_0 is rejected if $AG > \chi^2_\alpha(k-1)$ where $\chi^2_\alpha(k-1)$ is $100(1-\alpha)$ percentile of the chi-square distribution with k-1 degrees of freedom.

2.3 James second order method

The James test statistic for testing (1) is

$$J = \sum\nolimits_{i=1}^{k} T_i^2.$$

Therefore, H_0 is rejected if $W > c$ where,

$$\begin{aligned} c = {} & q + 0.5(3C_4 + C_2) \sum\nolimits_{i=1}^{k} ((1 - w_i/Q)^2 \Big/ (n_i - 1)) \\ & + (1/16)(3C_4 + C_2)^2 (1 - (k-3)/q) \{ \sum\nolimits_{i=1}^{k} ((1 - w_i/Q)^2 \Big/ (n_i - 1)) \}^2 \\ & + (1/2)(3C_4 + C_2) \{ (8R_{23} - 10R_{22} + 4R_{21} - 6R_{12}^2 + 8R_{12}R_{11} \\ & - 4R_{11}^2) + (2R_{23} - 4R_{22} + 2R_{21} - 2R_{12}^2 + 4R_{12}R_{11} - 2R_{11}^2) \\ & \times (C_2 - 1)(1/4)(-R_{12}^2 + 4R_{12}R_{11} - 2R_{12}R_{10} - 4R_{11}^2 R_{10} - R_{10}^2)(3C_4 - 2C_2 - 1) \} \\ & + (R_{23} - 3R_{22} + 3R_{21} - R_{20})(5C_6 + 2C_4 + C_2) + (3/16)(R_{12}^2 - 4R_{23} \\ & + 6R_{22} - 4R_{12} + R_{20})(35C_8 + 15C_6 + 9C_4 + 5C_2) + (1/16)(-2R_{22} + 4R_{12} \\ & - R_{20} + 2R_{12}R_{10} - 4R_{11}R_{10} + R_{10}^2)(9C_8 - 3C_6 - 5C_4 - C_2) \\ & + (1/4)(-R_{22} + R_{11}^2)(27C_8 + 3C_6 + C_4 + C_2) \\ & + (1/4)(R_{23} - R_{12}R_{11})(45C_8 + 9C_6 + 7C_4 + 3C_2), \end{aligned}$$

and for any positive integers a and b,

$$R_{ab} = \frac{1}{Q^b} \sum\nolimits_{i=1}^{k} \frac{w_i^b}{(n_i - 1)^a}, \qquad C_{2a} = \frac{q^a}{(k-1)(k+1)....(k+2a-3)},$$

and $q = \chi_\alpha^2(k-1)$.

2.4 The CAT

Pal et al. (2007) introduced the CAT in a general setup. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a population with the known density function $f(x|\lambda)$ with $\lambda = (\theta, \psi)$, where θ is the parameter of interest and ψ is the nuisance parameter. To test $H_0 : \theta = \theta_0$ against a suitable alternative H_1 at level α, the general methodology of the CAT for testing is given through the following steps:

1) First obtain $\hat{\lambda} = (\hat{\theta}, \hat{\psi})$, the MLE of λ.

2) Assume that H_0 is true, i.e., set $\theta = \theta_0$. Then find the MLE of ψ from the data. Call this as the restricted MLE of ψ under H_0 and denote by $\hat{\psi_{RML}}$.

3) Generate artificial sample $Y_1, Y_2, \cdots, Y_n$ from $f(x|\theta_0, \psi_{RML})$ a large number of times (say, M times). For each of these replicated samples, recalculate the MLE of $\lambda = (\theta, \psi)$. Retain only the component that is relevant for θ. Let these recalculated MLE values of θ be $\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_M$.

4) Let $\hat{\theta}_{(1)} < \hat{\theta}_{(2)} < .... < \hat{\theta}_{(M)}$ be the ordered values of $\hat{\theta}_{(\ell)}$, $1 \leq l \leq M$.

5) (i) For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$, define $\hat{\theta}_L = \hat{\theta}_{(\alpha M)}$. Reject H_0 if $\hat{\theta} < \hat{\theta}_L$ and accept H_0 otherwise. Alternatively, calculate the p-value as

$$p = \frac{1}{M} \sum_{l=1}^{M} I(\hat{\theta}_l < \hat{\theta}).$$

(ii) For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$, define $\hat{\theta}_U = \hat{\theta}_{((1-\alpha)M)}$. Reject H_0 if $\hat{\theta} > \hat{\theta}_U$ and accept H_0 otherwise. Alternatively, calculate the p-value as

$$p = \frac{1}{M}\sum_{l=1}^{M} I(\hat{\theta}_l > \hat{\theta}).$$

(iii) For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$, define $\hat{\theta}_L = \hat{\theta}_{((\frac{\alpha}{2})M)}$ and $\hat{\theta}_U = \hat{\theta}_{((1-\frac{\alpha}{2})M)}$. Reject H_0 if $\hat{\theta} < \hat{\theta}_L$ or $\hat{\theta} > \hat{\theta}_U$ and accept H_0 otherwise. Alternatively, calculate the p-value as:

$$p = 2\min(p_1, 1 - p_1),$$

where $p_1 = \frac{1}{M}\sum_{l=1}^{M} I(\hat{\theta}_l < \hat{\theta})$

Now implement the CAT for testing the equality of several log-normal means. To apply our proposed CAT, we first need to express H_0 in term of a suitable scalar θ. Define

$$\theta = h(\mu_i; \sigma_i^2) = \sum_{i=1}^{k} \frac{1}{v_i}(\eta_i - \bar{\eta})^2 = \sum_{i=1}^{k} \frac{\eta_i^2}{v_i} - \frac{\left(\sum_{i=1}^{k} \frac{\eta_i}{v_i}\right)^2}{\sum_{i=1}^{k} \frac{1}{v_i}},$$

where $\bar{\eta} = (\sum_{i=1}^{k} \frac{1}{v_i})^{-1} \sum_{i=1}^{k} \frac{\eta_i}{v_i}$. It is seen that the hypothesis in (2) is equivalent to

$$H_0^* : \theta = 0 \qquad vs \qquad H_A^* : \theta > 0.$$

If we apply the steps of CAT, then we have the following steps for testing the equality of means of several log-normal distributions:

1) Obtain $\hat{\mu}_i = \bar{X}_i$ and $\hat{\sigma}_i^2 = S_i^2, \quad i = 1, \cdots, k$ and calculate $\hat{\theta} = h(\hat{\mu}_i; \hat{\sigma}_i^2)$.

2) Assume that H_0 in (2) is true, i.e., set $\mu_i = \eta - \frac{1}{2}\sigma_i^2, 1 \leq i \leq k$. The likelihood function (underH_0) is a function of $(\eta, \sigma_1^2, ..., \sigma_k^2)$ only. The restricted MLEs of $\eta, \sigma_1^2, ..., \sigma_k^)$ denoted by $\hat{\eta}_{RML}$, $\hat{\sigma}^2_{i(RML)}$, $i = 1, \cdots, k$, are found using numerical methods, see Gill (2004). Define $\hat{\mu}_{i(RML)} = \hat{\eta}_{(RML)} - \frac{1}{2}\sigma^2_{i(RML)}$.

3) Generate artificial sample $Y_{i1}, ..., Y_{in_i} (= Y_i, say)$ independent random sample from $N(\hat{\mu}_{i(RML)}, \sigma^2_{i(RML)})$. Repeat this process M times. In the l-th replication $1 \leq l \leq M$ based on $Y_i^{(l)}$ get the MLEs of μ_i and σ_i^2 by (3) and call them as $\hat{\mu}_{0i}^{(l)}$ and $\hat{\sigma}_{0i}^{2(l)}$. Then recalculate $\hat{\theta}$ as $\hat{\theta}_{0l} = h(\hat{\mu}_{0i}^{(l)}; \hat{\sigma}_{0i}^{2(l)})$.

4) Order the $\hat{\theta}_{0l}$ values as $\hat{\theta}_{0(1)} \leq \hat{\theta}_{0(2)} \leq ... \leq \hat{\theta}_{0(M)}$.

5) Let $\hat{\theta}_U = \hat{\theta}_{0((1-\alpha)M)}$ and reject H_0 if $\hat{\theta} > \hat{\theta}_U$ and accept H_0 otherwise. Alternatively, if the value $p = \frac{1}{M}\sum_{l=1}^{M} I(\hat{\theta}_{0l} > \hat{\theta}_{ML})$ is smaller than the nominal level α, then reject H_0.

3 Simulation studies and numerical example

In this section, first, a simulation study is performed to evaluate the size and power properties of the CAT. This method is compared with the three methods provided by Guo and Luh (2000). Then we provide a real data analysis for testing equality means by our proposed method and existing methods.

3.1 Simulation studies

We performed a simulation study in order to compare the actual sizes and powers of the James, Welch, Alexander- Govern tests and CAT with M=10,000 replications for testing equality of k log-normal distributions. For this purpose, the random samples with size n_i is generated from log-normal distribution with the parameters μ_i and σ_i^2 $(i = 1, ..., k)$. We considered the cases $k = 2, 3, 4, 5, 6$.

Guo and Luh (2000) in their simulation study considered six sample size patterns. Also, they consider twelve designs were then totally crossed by the sample size pattern which made 24 configurations. The number of their replications was 10,000. Their simulation was for large sample sizes, but we also consider small sample sizes. We consider the 24 configurations of their simulation and another designs and sample size patterns, also the number of our replications is 20,000. For comparing the actual sizes, the values are chosen such that the means of the log-normal distributions are equal.

We considered two cases for the common parameter η (logarithm of common log-normal mean) for comparing actual sizes of the methods. The results of the actual sizes for $\eta = 2$ are given in Table 1, and for $\eta = 0.5$ are given in Table 2. We conclude that the CAT has a type I error probability very close to the nominal level for all cases. However, the other methods are more liberal (type I error probability is larger than the nominal level) even for large samples. Only, the actual sizes of the James, Welch and Alexander- Govern tests are close to the nominal level when σ_i^2s are close to each other.

For power comparison, in the designs of equal variance, the mean value of the first group is added 1. In addition, we consider other designs. The results are given in Table 3. It can conclude that the power of CAT can be as good as (if not better than) the other methods when the σ_i^2s are close to each other. But the powers of James, Welch and Alexander- Govern tests are larger than the power CAT.

Table 1: The actual sizes of the tests at nominal level 0.05 with common $\eta = 2$

$\mu_1, \cdots, \mu_k$	$\sigma_1^2, \cdots, \sigma_k^2$	$n_1, \cdots, n_k$	James	Welch	A-G	CAT
1.5,-2.5	1,9	10,10	0.0891	0.0892	0.0887	0.0503
1.5,-2.5	1,9	15,10	0.0921	0.0921	0.0919	0.0491
1,-2	2,8	10,10	0.0774	0.0771	0.0752	0.0492
1,-2	2,8	15,10	0.0817	0.0820	0.0811	0.0498
1.5,1.5	1,1	30,30	0.0459	0.0459	0.0451	0.0498
1.5,1.5	1,1	40,20	0.0465	0.0465	0.0458	0.0502
1.5,0	1,4	30,30	0.0518	0.0518	0.0515	0.0481
1.5,0	1,4	40,20	0.0618	0.0619	0.0617	0.0504
1.5,-0.5	1,5	30,30	0.0546	0.0546	0.0541	0.0480
1.5,-0.5	1,5	40,20	0.0641	0.0642	0.0640	0.0500
1.5,-2.5	1,9	30,30	0.0611	0.0612	0.0611	0.0488
1.5,-2.5	1,9	40,20	0.0733	0.0733	0.0732	0.0498
0.5,0,-.0.5	1,2,3	10,10,10	0.0512	0.0510	0.0522	0.0461
0.5,0,-.0.5	1,2,3	15,10,10	0.0613	0.0614	0.0630	0.0468
2,-1.5,1	2,9,4	10,10,10	0.0751	0.0749	0.0735	0.0453
2,-1.5,1	2,9,4	15,10,10	0.0883	0.0887	0.0895	0.0474
0.5,0,-0.5,-1	1,2,3,4	10,10,10,10	0.0606	0.0614	0.0613	0.0389
0.5,0,-0.5,-1	1,2,3,4	15,10,10,15	0.0679	0.0688	0.0702	0.0457
1,0,-2.5,-0.5	2,4,9,5	10,10,10,10	0.0692	0.0702	0.0707	0.0417
1,0,-2.5,-0.5	2,4,9,5	15,10,10,15	0.0796	0.0806	0.0798	0.0453
1.5,1.5,1.5,1.5	1,1,1,1	30,30,30,30	0.0442	0.0442	0.0442	0.0467
1.5,1.5,1.5,1.5	1,1,1,1	45,35,25,15	0.0494	0.0498	0.0500	0.0493
1.5,1.5,1.5,0	1,1,1,4	30,30,30,30	0.0515	0.0516	0.0512	0.0463
1.5,1.5,1.5,0	1,1,1,4	45,35,25,15	0.0651	0.0655	0.0630	0.0481
1.5,1.5,1.5,-0.5	1,1,1,5	30,30,30,30	0.0561	0.0561	0.0556	0.0503
1.5,1.5,1.5,-0.5	1,1,1,5	45,35,25,15	0.0659	0.0666	0.0646	0.0491
1.5,1.5,1.5,-2.5	1,1,1,9	30,30,30,30	0.0604	0.0605	0.0600	0.0487
1.5,1.5,1.5,-2.5	1,1,1,9	45,35,25,15	0.0746	0.0752	0.0715	0.0500
0.5,0,-0.5,-1,-1.5	1,2,3,4,5	10,10,10,10,10	0.0742	0.0764	0.0742	0.0394
0.5,0,-0.5,-1,-1.5	1,2,3,4,5	15,15,10,10,15	0.0824	0.0842	0.0816	0.0491
1,0,0.25,-0.5,-2.5	2,4,3.5,5,9	10,10,10,10,10	0.0677	0.0698	0.0696	0.0381
1,0,0.25,-0.5,-2.5	2,4,3.5,5,9	15,15,10,10,15	0.0725	0.0741	0.0750	0.0440
1,1,-0.5,-2.5,1,1	2,2,5,9,2,2	10,10,10,10,10,10	0.0725	0.0743	0.0706	0.0413
1,1,-0.5,-2.5,1,1	2,2,5,9,2,2	15,10,15,10,15,10	0.0737	0.0766	0.0727	0.0459
1,0,-0.5,-2.5,0.25,1	2,4,5,9,3.5,2	10,10,10,10,10,10	0.0693	0.0724	0.0703	0.0366
1,0,-0.5,-2.5,0.25,1	2,4,5,9,3.5,2	15,10,15,10,15,10	0.0711	0.0736	0.0709	0.0400
1.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	30,30,30,30,30,30	0.0474	0.0481	0.0479	0.0488
1.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	45,35,35,25,25,15	0.0525	0.0534	0.0532	0.0508
1.5,1.5,1.5,1.5,1.5,0	1,1,1,1,1,4	30,30,30,30,30,30	0.0550	0.0554	0.0553	0.0500
1.5,1.5,1.5,1.5,1.5,0	1,1,1,1,1,4	45,35,35,25,25,15	0.0623	0.0630	0.0598	0.0492
1.5,1.5,1.5,1.5,1.5,-0.5	1,1,1,1,1,5	30,30,30,30,30,30	0.0552	0.0558	0.0549	0.0485
1.5,1.5,1.5,1.5,1.5,-0.5	1,1,1,1,1,5	45,35,35,25,25,15	0.0658	0.0667	0.0624	0.0486
1.5,1.5,1.5,1.5,1.5,-2.5	1,1,1,1,1,9	30,30,30,30,30,30	0.0554	0.0558	0.0553	0.0459
1.5,1.5,1.5,1.5,1.5,-2.5	1,1,1,1,1,9	45,35,35,25,25,15	0.0717	0.0730	0.0682	0.0488

Table 2: The actual sizes of the tests at nominal level 0.05 with common $\eta = 0.5$

$\mu_1, \cdots, \mu_k$	$\sigma_1^2, \cdots, \sigma_k^2$	$n_1, \cdots, n_k$	James	Welch	A-G	CAT
0,-4	1,9	10,10	0.0865	0.0871	0.0870	0.0489
0,-4	1,9	15,10	0.0905	0.0908	0.0907	0.0506
-0.5,-3.5	2,8	10,10	0.0748	0.0763	0.0770	0.0503
-0.5,-3.5	2,8	15,10	0.0850	0.0859	0.0856	0.0513
0,0	1,1	30,30	0.0437	0.0446	0.0448	0.0483
0,0	1,1	40,20	0.0444	0.0455	0.0456	0.0478
0, -1.5	1,4	30,30	0.0531	0.0532	0.0532	0.0496
0,-1.5	1,4	40,20	0.0593	0.0595	0.0595	0.0484
0,-2	1,5	30,30	0.0569	0.0571	0.0571	0.0505
0,-2	1,5	40,20	0.0675	0.0677	0.0676	0.0513
0,-4	1,9	30,30	0.0627	0.0628	0.0627	0.0483
0,-4	1,9	40,20	0.0723	0.0724	0.0723	0.0507
0,-0.5,-1	1,2,3	10,10,10	0.0547	0.0536	0.0540	0.0451
0,-0.5,-1	1,2,3	15,10,10	0.0648	0.0632	0.0627	0.0480
-0.5,-4,-1.5	2,9,4	10,10,10	0.0677	0.0692	0.0694	0.0435
-0.5,-4,-1.5	2,9,4	15,10,10	0.0868	0.0870	0.0864	0.0472
0,-0.5,-1,-1.5	1,2,3,4	10,10,10,10	0.0606	0.0602	0.0594	0.0429
0,-0.5,-1,-1.5	1,2,3,4	15,10,10,15	0.0694	0.0674	0.0664	0.0455
-0.5,-1.5,-4,-2	2,4,9,5	10,10,10,10	0.0682	0.0683	0.0678	0.0414
-0.5,-1.5,-4,-2	2,4,9,5	15,10,10,15	0.0825	0.0817	0.0809	0.0465
0,0,0,0	1,1,1,1	30,30,30,30	0.0447	0.0449	0.0449	0.0468
0,0,0,0	1,1,1,1	45,35,25,15	0.0486	0.0485	0.0482	0.0485
0,0,0,-1.5	1,1,1,4	30,30,30,30	0.0504	0.0507	0.0505	0.0462
0,0,0,-1.5	1,1,1,4	45,35,25,15	0.0645	0.0668	0.0661	0.0500
0,0,0,-2	1,1,1,5	30,30,30,30	0.0574	0.0582	0.0580	0.0506
0,0,0,-2	1,1,1,5	45,35,25,15	0.0655	0.0687	0.0682	0.0505
0,0,0,-4	1,1,1,9	30,30,30,30	0.0605	0.0620	0.0619	0.0506
0,0,0,-4	1,1,1,9	45,35,25,15	0.0704	0.0746	0.0740	0.0486
0,-0.5,-1,-1.5,-2	1,2,3,4,5	10,10,10,10,10	0.0694	0.0718	0.0692	0.0405
0,-0.5,-1,-1.5,-2	1,2,3,4,5	15,15,10,10,15	0.0784	0.0794	0.0773	0.0469
-0.5,-1.5,-1.25,-2,-4	2,4,3.5,5,9	10,10,10,10,10	0.0682	0.0693	0.0674	0.0428
-0.5,-1.5,-1.25,-2,-4	2,4,3.5,5,9	15,15,10,10,15	0.0728	0.0724	0.0710	0.0446
-0.5,-0.5,-2,-4,-0.5,-0.5	2,2,5,9,2,2	10,10,10,10,10,10	0.0714	0.0761	0.0730	0.0434
-0.5,-0.5,-2,-4,-0.5,-0.5	2,2,5,9,2,2	15,10,15,10,15,10	0.0707	0.0752	0.0726	0.0440
-0.5,-1.5,-2,-4,-1.25,-0.5	2,4,5,9,3.5,2	10,10,10,10,10,10	0.0726	0.0737	0.0711	0.0394
-0.5,-1.5,-2,-4,-1.25,-0.5	2,4,5,9,3.5,2	15,10,15,10,15,10	0.0764	0.0804	0.0778	0.0468
0,0,0,0,0,0	1,1,1,1,1,1	30,30,30,30,30,30	0.0478	0.0477	0.0474	0.0486
0,0,0,0,0,0	1,1,1,1,1,1	45,35,35,25,25,15	0.0516	0.0522	0.0515	0.0497
0,0,0,0,0,-1.5	1,1,1,1,1,4	30,30,30,30,30,30	0.0524	0.0526	0.0524	0.0471
0,0,0,0,0,-1.5	1,1,1,1,1,4	45,35,35,25,25,15	0.0608	0.0643	0.0633	0.0498
0,0,0,0,0,-2	1,1,1,1,1,5	30,30,30,30,30,30	0.0525	0.0531	0.0527	0.0471
0,0,0,0,0,-2	1,1,1,1,1,5	45,35,35,25,25,15	0.0603	0.0642	0.0632	0.0491
0,0,0,0,0,-4	1,1,1,1,1,9	30,30,30,30,30,30	0.0584	0.0588	0.0583	0.0474
0,0,0,0,0,-4	1,1,1,1,1,9	45,35,35,25,25,15	0.0692	0.0749	0.0737	0.0501

Table 3: Empirical powers of the tests

$\mu_1, \cdots, \mu_k$	$\sigma_1^2, \cdots, \sigma_k^2$	$n_1, \cdots, n_k$	James	Welch	A-G	CAT
2.5,1.5	1,1	10,10	0.3902	0.3869	0.3770	0.3982
2.5,1.5	1,1	15,10	0.4825	0.4810	0.4741	0.4619
2.5,1.5	1,1	30,30	0.8679	0.8677	0.8666	0.8769
2.5,1.5	1,1	40,20	0.7926	0.7930	0.7911	0.7921
1.5,1.5	0.5,1	15,10	0.0433	0.0437	0.0418	0.0488
1.5,1.5	0.5,1	30,30	0.1270	0.1271	0.1258	0.1285
1,-4	1,9	10,10	0.1581	0.1587	0.1586	0.0900
1,-4	1,9	15,10	0.1682	0.1683	0.1682	0.0964
1,-4	1,9	30,30	0.1862	0.1863	0.1863	0.1532
0.5,-3.5	2,8	10,10	0.1508	0.1527	0.1530	0.0970
0.5,-3.5	2,8	15,10	0.1554	0.1565	0.1562	0.0945
0.5,-3.5	2,8	30,30	0.1904	0.1912	0.1911	0.1653
2,1,2	1.5,1,2	15,10,10	0.4866	0.4864	0.4875	0.4906
2,1,2	1.5,1,2	30,30,30	0.9575	0.9575	0.9575	0.9619
1,-0.5,-1	1,2,3	10,10,10	0.3073	0.3082	0.3090	0.2225
1,-0.5,-1	1,2,3	15,10,10	0.3694	0.3622	0.3610	0.2524
0.5,-4,-1.5	2,9,4	10,10,10	0.1769	0.1783	0.1780	0.0940
0.5,-4,-1.5	2,9,4	15,10,10	0.2249	0.2214	0.2202	0.1068
2.5,1.5,1.5,1.5	1,1,1,1	10,10,10,10	0.3654	0.3669	0.3462	0.3645
2.5,1.5,1.5,1.5	1,1,1,1	15,10,10,15	0.5352	0.5382	0.5424	0.5152
2.5,1.5,1.5,1.5	1,1,1,1	30,30,30,30	0.9250	0.925	0.9231	0.9296
2.5,1.5,1.5,1.5	1,1,1,1	45,35,25,15	0.9631	0.9633	0.9671	0.9583
1,0.75,0.5,1	1.5,1,1.4,1.2	15,10,10,15	0.1221	0.1231	0.1241	0.1177
1,0.75,0.5,1	1.5,1,1.4,1.2	30,30,30,30	0.2369	0.2372	0.2369	0.2466
1,0,0,-2	1,1,1,5	10,10,10,10	0.3391	0.3476	0.3449	0.2252
1,0,0,-2	1,1,1,5	30,30,30,30	0.8853	0.8866	0.8862	0.8637
1,0,0,-2	1,1,1,5	45,35,25,15	0.9488	0.9406	0.9393	0.8772
1,0,0,-2	1,1,1,5	10,10,10,10	0.3366	0.3485	0.3451	0.2221
1,0,0,-4	1,1,1,9	30,30,30,30	0.8843	0.8844	0.8843	0.8438
1,0,0,-4	1,1,1,9	45,35,25,15	0.9464	0.9376	0.9369	0.8446
3,3.5,3.5,3,4	1.5,1,1.75,1.25,2	15,15,10,10,15	0.3005	0.3065	0.3101	0.2864
3,3.5,3.5,3,4	1.5,1,1.75,1.25,2	30,30,30,30,30	0.7647	0.7652	0.7588	0.7728
0.5,-0.5,-2,-4,-0.5,-0.5	2,2,5,9,2,2	10,10,10,10,10,10	0.1494	0.1586	0.1517	0.0789
0.5,-0.5,-2,-4,-0.5,-0.5	2,2,5,9,2,2	15,10,15,10,15,10	0.2131	0.2142	0.2069	0.1009
0.5,-1.5,-2,-4,-1.25,-0.5	2,4,5,9,3.5,2	10,10,10,10,10,10	0.1663	0.1732	0.1670	0.0842
0.5,-1.5,-2,-4,-1.25,-0.5	2,4,5,9,3.5,2	15,10,15,10,15,10	0.2308	0.2307	0.2236	0.1093
2.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	10,10,10,10,10,10	0.3179	0.3272	0.2942	0.3125
2.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	15,10,15,10,15,10	0.4878	0.4982	0.4894	0.4654
2.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	30,30,30,30,30,30	0.9159	0.9165	0.9056	0.9210
2.5,1.5,1.5,1.5,1.5,1.5	1,1,1,1,1,1	45,35,35,25,25,15	0.9775	0.9780	0.9792	0.9749
1,1,0,1.25,1,1.5	1.5,1,1.4,1.2,1.3,1	15,10,15,10,15,10	0.3593	0.3666	0.3689	0.3503
1,1,0,1.25,1,1.5	1.5,1,1.4,1.2,1.3,1	30,30,30,30,30,30	0.7906	0.7913	0.7862	0.7944

3.2 A real example

In this section, we analyze pharmacokinetics data from alcohol interaction in men that have been studied by Bradstreet and Liss (1995). This data also is analyzed by Tian and Wu (2007). The summary statistics of the log-transformed data is presented in Table 4. We are interesting to test the equality of means of the three groups. The test

Table 4: The summary statistics for log-transformed pharmacokinetics data

Group	n_i	$\bar{y}_i$	S_i^2
1	22	2.601	0.24
2	22	2.596	0.20
3	22	2.599	0.17

statistics of the James, Welch and Alexander-Govern methods based on the above data are given by 0.0668, 0.0329, and 0.0651, respectively. Also, their corresponding critical values for $\alpha = 0.05$ are 6.5368, 3.2213 and 5.9915. Also, the p-value of the CAT based on the above data is 0.9698 with M=10,000. Therefore, none of the tests rejects the hypothesis of equality of means for the three groups.

4 Conclusions

In simulation study, we compared the performance of the James, Welch, Alexander-Govern tests and CAT for one-way fixed effects ANOVA models of log-normal in terms of actual size and empirical power. The actual size of the CAT is close to the nominal level in all cases and usually is smaller than the actual size of other tests. In terms of power, the CAT is satisfactory. In addition, the CAT without going through the analytical derivation of the relevant test does the testing automatically.

References

Alexander, R.A. and Govern, D.M. (1994). A new and simpler approximation for anova under variance heterogeneity. *Journal of Educational Statistics*, **19**(2), 91–101.

Bradstreet, T.E. and Liss, C.L. (1995). Favorite data sets from early (and late) phases of drug research-part 4. In *ASA Proceedings of the Section on Statistical Education*, Volume 335, Page 340.

Chang, C.H. and Pal, N. (2008). A revisit to the behrens–fisher problem: comparison of five test methods. *Communications in Statistic-Simulation and Computation*, **37**(6), 1064–1085.

Chang, C.H., Pal, N., Lim, W.K., and Lin, J.J. (2010). Comparing several population means: a parametric bootstrap method, and its comparison with usual anova f test as well as anom. *Computational Statistics*, **25**(1), 71–95.

Crow, E.L. and Shimizu, K. (1987). *Lognormal distributions.* Marcel Dekker New York.

Gill, P. S. (2004). Small-sample inference for the comparison of means of log-normal distributions. *Biometrics*, **60**(2), 525–527.

Gökpınar, E. Y. and Gökpınar, F. (2012). A test based on the computational approach for equality of means under the unequal variance assumption abstract— full text hacettepe journal of mathematics and statistics. *Hacettepe journal of Mathematics and Statistics*, **41**(4), 605–613.

Guo, J.H. and Luh, W.M. (2000). Testing methods for the one-way fixed effects anova models of log-normal samples. *Journal of Applied Statistics*, **27**(6), 731–738.

James, G. (1951). The comparison of several groups of observations when the ratios of the population variances are unknown. *Biometrika*, **38**(3/4), 324–329.

Johnson, N.L., Kotz, S. and Balakrishnan, N. (1994). Continuous univariate distributions, vol. 1 of wiley series in probability and statistics.

Pal, N., Lim, W.K. and Ling, C.H. (2007). A computational approach to statistical inferences. *Journal of Applied Probability & Statistics*, **2**(1), 13–35.

Tian, L. and Wu, J. (2007). Inferences on the common mean of several log-normal populations: The generalized variable approach. *Biometrical Journal: Journal of Mathematical Methods in Biosciences*, **49**(6), 944–951.

Welch, B.L. (1951). On the comparison of several mean values: an alternative approach. *Biometrika*, **38**(3/4), 330–336.