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On construction confidence interval for linear combination means of several heterogeneous log-normal distributions

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Abstract: We consider the problem of constructing confidence interval for linear combination of the means of several log-normal distributions. We apply the generalized confidence interval (GCI) approach and the method of variance estimate recovery (MOVER) to construct confidence intervals for the linear combination of log-normal means. We then compare the performances of the proposed confidence intervals via a simulation study and a real data example. Simulation results show that our proposed MOVER and GCI confidence intervals can be recommended generally for different sample sizes and different number of groups.

Keywords: Coverage probability; Generalized confidence interval; Log-normal distribution; Method of variance estimate recovery; Monte Carlo simulation.
Mathematics Subject Classification (2010): 99X99, 99X99.

1 Introduction

In practice such as biology, economy, insurance, medicine and pharmacology, a very applicable distribution for positive right-skewed data is the log-normal distribution; see Lacey et al. (1997), Julius and Debarnot (2000), Shen et al. (2006) and references therein. As an example, Tian and Wu (2007) consider the medical charge data from analyzed this data set and concluded that the African American group (119 patients) and the white group (106 patients) are distributed as a log-normal distribution. Also, Keshavarzi et al. (2011) studied a possible link between the epidemiologic patterns of esophageal cancer (EC) and the anomalous concentration of some ions and elements in the drinking water sources in the four regions: Gonbad, Dashlibroon, Maravetappeh and Gorgan all in North of Iran. Primary analysis showed that the log-transformed data are normally distributed in all regions. The log-normal distribution is especially used

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in biomedical research. As noted by Krishnamoorthy and Oral (2015), the growth rates of soft tissue metastases of breast cancer can be modelled by a log-normal distribution and the transcript levels of five different genes in individual cells from mouse pancreatic islets are log-normally distributed. For more examples and applications of log-normal distribution in epidemiological and health related studies see Lacey et al. (1997) and Lin and lee (2005).

Suppose that there are k independent samples that are taken from k populations and each population follows a log-normal distribution. In practice, it is firstly of interest to test the hypothesis of the equality of log-normal means. One can refer to Gokpinara and Gokpinara (2015), Jafari and Abdollahnezhad (2015), Krishnamoorthy and Oral (2015) and li (2009); among others, for the problem of testing the equality of several heterogeneous log-normal means. When the hypothesis of log-normal means equality is done, one of the problems of interest is to make inferences on the linear combination of the mean of populations, like common mean, differences, contrasts, and etc. It is worth mentioning that several researchers have considered the problem of making inferences on the linear combination of the mean of several normal populations; among others, see Lin and Lee (2005) and the references therein. However, there are situations that the log-transformed observations have a normal distribution instead of the original observations; i.e. the observations have a log-normal distribution. Although by the logarithm transformation the problem backs to the normal case, a research may interest in making inference about the mean response in the original scales since the mean of log-transformed data may not have a practical interpretation; see Tian and Wu (2007) and Zhou et al. (1997). For example, in the example of Keshavarzi et al. (2011), they considered Gorgan as a case–control point due to its comparatively low incidence rate of EC and Gonbad, Dashlibroon and Maravetappeh as high esophageal cancer areas with a common mean. It is of interest in making inferences about the anomalous concentration of some ions in the drinking water sources in the three regions with comparison of Gorgan region.

In this paper, we apply the generalized pivotal quantity (GPQ) approach to construct a confidence interval for the linear combination mean of several heterogeneous log-normal distributions. Also, we proposed a closed-form approximate confidence based on MOVER approach. Note that a test for the linear combination mean can be performed by inverting the confidence interval. Then, we compare the proposed confidence intervals via a simulation study in terms of coverage probability (CP) and average length (AL). Simulation results show that our proposed MOVER confidence intervals is simple to implement and is preferred generally for small and moderate sample sizes. Also, GCI is appropriate for large sample sizes.

2 Notation and necessary background

Suppose that X_{ij} , $i = 1, \cdots , k$, $j = 1, \cdots , n_i$ be random samples from k independent populations where i -th population has a log-normal distribution with the parameters μ_i and σ_i^2 , denoted by $\log(\mu_i, \sigma_i^2)$, $i = 1, \cdots , k$. Let $\eta_i = E(X_{ij}) = \exp(\mu_i + \frac{\sigma_i^2}{2})$ denote the mean of the i -th population. We are interested in constructing confidence interval for $\eta = \sum_{i=1}^{k} c_i\eta_i$ where c_i 's are known values as differences, contrasts or etc.

Let $Y_{ij} = \log(X_{ij})$, $i = 1, \cdots , k$ and $j = 1, \cdots , n_i$ It is well known that the Y_{ij}

is distributed as a normal distribution with the mean μ_i and variance σ_i^2 , henceforth $N(\mu_i, \sigma_i^2)$. The unbiased maximum likelihood estimators and uniformly minimum variance unbiased estimators (UMVUEs) of μ_i and σ_i^2 can be obtained by the following equations $\bar{Y}_i = \sum_{j=1}^{n_i} \frac{Y_{ij}}{n_i}$ and $S_i^2 = \frac{1}{n_i-1} \sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_i)^2$. It is well-known that $\bar{Y}_i$ and S_i^2 are independent (Sadooghi-Alvandi and Malekzadeh, 2014), and $\bar{Y}_i \sim N(\mu_i, \frac{\sigma_i^2}{n_i})$ and $\frac{(n_i-1)S_i^2}{\sigma_i^2} \sim \chi^2_{(n_i-1)}$, $i = 1, \cdots, k$ where $\chi^2_{(\tau)}$ denotes a chi-square distribution with τ degrees of freedom.

3 The proposed confidence intervals

In this section, we first provide a version of the approximated MOVER confidence interval and then a generalized confidence interval for the linear combination of the means of log-normal populations.

3.1 The MOVER confidence interval

The MOVER is a useful technique to find a closed-form approximate confidence interval for a linear combination of parameters based on the confidence intervals of the individual parameters; see Zou and Donner (2008), Zou et al. (2009a, b) and Krishnamoorthy and Oral (2015). To describe the MOVER technique, consider a linear combination $\sum_{i=1}^k w_i\theta_i$ of parameters $\theta_1, \cdots, \theta_k$ where w_i are known constants. Let $\hat{\theta}_i$'s be independent unbiased estimates of θ_i 's. In addition (L_i, U_i) , let denote the $100(1-\alpha)\%$ confidence interval for θ_i, $i = 1, \cdots, k$. Then, $100(1-\alpha)\%$ the MOVER confidence interval for $\sum_{i=1}^k w_i\theta_i$ is (U, L) where

$$L = \sum_{i=1}^k w_i\hat{\theta}_i - \sqrt{\sum_{i=1}^k w_i^2(\hat{\theta}_i - L_i^*)^2}; \quad L_i^* = \begin{cases} L_i, & w_i > 0, \\ U_i, & w_i < 0. \end{cases}$$

$$U = \sum_{i=1}^k w_i\hat{\theta}_i - \sqrt{\sum_{i=1}^k w_i^2(\hat{\theta}_i - U_i^*)^2}; \quad U_i^* = \begin{cases} U_i, & w_i > 0, \\ L_i, & w_i < 0. \end{cases}$$

In our problem, $\hat{\eta_i}^* = \bar{Y}_i + \frac{1}{2}S_i^2$ is an unbiased estimate of $\eta_i^* = \mu_i + \frac{1}{2}\sigma_i^2$. $i = 1, \cdots, k$ Therefore, a confidence interval can be obtained for the log-normal mean η_i^* by combining the individual confidence intervals for μ_i and σ_i^2 . In this regard, notice that the exact confidence intervals for μ_i and σ_i^2 are given by Abdel-Karim (2015) as $(\bar{Y}_i + \frac{t_{(1-\frac{\alpha}{2})}(n_i-1)S_i}{\sqrt{n_i}})$ and $(\frac{(n_i-1)S_i^2}{\chi^2_{1-\frac{\alpha}{2}}(n_i-1)}, \frac{(n_i-1)S_i^2}{\chi^2_{\frac{\alpha}{2}}(n_i-1)})$ where $t_\gamma(n)$ and $\chi^2_\gamma(n)$ denote the γ -th percentile of t -distribution and chi-square distribution with degrees of n freedom respectively. Using the MOVER, an approximate confidence interval for η_i^* can be (L_i^*, U_i^*) where

$$L_i^* = \hat{\eta_i}^* - s_i\sqrt{\frac{t^2_{\frac{\alpha}{2}}(n_i-1)}{n_i} + \frac{s_i^2}{4}(1 - \frac{n_i-1}{\chi^2_{1-\frac{\alpha}{2}}(n_i-1)})^2},$$

and

$$U_i^* = \hat{\eta_i}^* + s_i\sqrt{\frac{t^2_{\frac{\alpha}{2}}(n_i-1)}{n_i} + \frac{s_i^2}{4}(1 - \frac{n_i-1}{\chi^2_{\frac{\alpha}{2}}(n_i-1)})^2};$$

See Krishnamoorthy and Oral (2015). Therefore, the $100(1-\alpha)\%$ approximate confidence interval for the i th population mean, i.e. η_i , is (L_i, U_i) where $L_i = \exp(L_i^*)$ and $U_i = \exp(U_i^*)$. Also, the $100(1-\alpha)\%$ approximate confidence interval for η the can be (L, U) where

$$L = \sum_{i=1}^{k} c_i \exp(\hat{\eta_i}^*) - \sqrt{(c_i^2 \exp(\hat{\eta_i}^*) - L_i^{**})^2}; \quad L_i^{**} = \begin{cases} L_i, & c_i > 0, \\ U_i, & c_i < 0. \end{cases}$$

$$U = \sum_{i=1}^{k} c_i \exp(\hat{\eta_i}^*) - \sqrt{(c_i^2 \exp(\hat{\eta_i}^*) - U_i^{**})^2}; \quad U_i^{**} = \begin{cases} U_i, & c_i > 0, \\ L_i, & c_i < 0. \end{cases}$$

Remark 3.1. *The null hypothesis $H_0 : \eta = \eta_0$ at level α is rejected when $(L - \eta_0) \times (U - \eta_0) > 0$.*

3.2 The generalized confidence interval

The concepts of generalized test variable and generalized pivotal quantity (GPQ) are introduced by Tsui and Weerahandi (1989) and Weerahandi (1993), respectively, and are successfully applied to different problems. Here, we use this concept of generalized pivotal quantity for constructing a generalized confidence interval. Based on $\bar{y}_i$ and s_i as observed values of $\bar{Y}_i$ and S_i GPQ of η_i^* is given by

$$R_{\eta_i^*} = \bar{x}_i - Z_i s_i \sqrt{\frac{n_i-1}{n_i U_i}} + \frac{n_i-1}{2U_i},$$

for $i = 1, \cdots, k$, where Z_i and U_i are independent random variables distributed as $N(0,1)$ and $\chi^2_{(n_i-1)}$ respectively. The generalized pivotal quantity related to η_i is $R_i = \exp(R_i^*)$ for $i = 1, \cdots, k$. The generalized pivotal quantity for $\eta = \sum_{i=1}^{k} c_i \eta_i$ is given by $R_i = \sum_{i=1}^{k} c_i R_i$ and the generalized confidence interval at confidence level $100(1-\alpha)\%$ for η is

$$(R_{\frac{\alpha}{2}}, R_{1-\frac{\alpha}{2}}),$$

Where R_β is a β -th percentile of the GPQ R. It can be estimated easily by the following algorithm:

(i) Compute $\bar{y}_i$'s and s_i 's from the data.

(ii) Generate Z_i and U_i and compute $R^*_{\eta_i}$ given in (2) and subsequent $R = \sum_{i=1}^{k} c_i R_i$.

(iii) Repeat Step (ii) M times to obtain M realizations of R. Calculate the empirical $(1-\alpha)$ -th percentile of generated R as an estimate of $R_{1-\alpha}$.

Remark 3.2. *The null hypothesis $H_0 : \eta = \eta_0$ at level α is rejected when the generalized p-value*

$$p_G = 2\min(P(R > \eta_0), P(R \leq \eta_0)),$$

is less than α.

Table 1: The estimated coverage probability and average length for $k = 3$ populations.

n_1, n_2, n_3	$\sigma_1^2, \sigma_2^2, \sigma_3^2$	$c_1 = (1, 1, -2)'$				$c_2 = (0.5, -2, 1.5)'$			
		GCI		MOVER		GCI		MOVER	
		CP	AL	CP	AL	CP	AL	CP	AL
5,5,5	0.1,0.1,0.1	0.9756	25.2426	0.9864	28.4155	0.9759	25.9945	0.9895	27.9745
	0.1,0.5,0.1	0.9767	1326.702	0.9913	1143.724	0.9686	1908.063	0.9872	1603.104
	1,0.75,0.5	0.9702	2.8460e8	0.9866	1.4173e8	0.9712	1.4661e8	0.9985	1.3766e8
	0.75,1,1.5	0.9677	1.5552e14	0.9803	4.3044e13	0.9722	6.2901e19	0.9949	1.4585e20
	0.5,1.5,1	0.9727	3.5034e12	0.9865	6.1443e12	0.9680	1.0551e12	0.9928	5.8779e11
	0.1,0.5,0.75	0.9678	1901573.4	0.9803	2920296.5	0.9721	51866.31	0.9931	108658.85
	0.5,1,0.25	0.9724	5.2951e8	0.9901	3.0108e8	0.9685	5.0167e10	0.9918	8.1934e9
10,8,10	0.1,0.1,0.1	0.9671	9.8874	0.9796	10.7369	0.9619	11.1302	0.9799	11.5220
	0.1,0.5,0.1	0.9629	25.0651	0.9812	24.2335	0.9563	44.8104	0.9744	42.3669
	1,0.75,0.5	0.9629	105.7108	0.9768	96.6527	0.9656	134.9410	0.9941	136.4065
	0.75,1,1.5	0.9605	495.1051	0.9748	742.4245	0.9670	596.1864	0.9949	781.8514
	0.5,1.5,1	0.9636	2240.659	0.9767	2037.639	0.9601	8537.120	0.9947	6819.945
	0.1,0.5,0.75	0.9629	63.9863	0.9791	89.1286	0.9609	69.4295	0.9916	82.7594
	0.5,1,0.25	0.9645	145.6557	0.9810	132.5060	0.9577	276.6461	0.9887	251.3103
10,15,10	0.1,0.1,0.1	0.9626	8.9579	0.9747	9.9542	0.9626	8.2818	0.9794	8.9668
	0.1,0.5,0.1	0.9652	12.6027	0.9794	13.1083	0.9516	18.5982	0.9681	18.4896
	1,0.75,0.5	0.9632	68.2992	0.9777	72.4810	0.9635	52.7637	0.9929	65.7712
	0.75,1,1.5	0.9629	396.9803	0.9779	681.3994	0.9603	293.8598	0.9898	485.8124
	0.5,1.5,1	0.9630	118.9784	0.9770	168.1819	0.9595	127.2376	0.9906	158.4371
	0.1,0.5,0.75	0.9573	52.9722	0.9754	81.1631	0.9589	45.6009	0.9852	64.0518
	0.5,1,0.25	0.9662	32.5156	0.9773	32.7835	0.9590	41.8454	0.9856	42.2639
10,15,20	0.1,0.1,0.1	0.9609	6.6965	0.9706	6.9774	0.9586	6.9023	0.9726	7.1375
	0.1,0.5,0.1	0.9587	10.9286	0.9714	10.8560	0.9524	17.7288	0.9657	17.3803
	1,0.75,0.5	0.9579	56.5311	0.9754	53.7609	0.9610	44.0315	0.9931	54.8130
	0.75,1,1.5	0.9605	67.0924	0.9741	83.0401	0.9602	62.9027	0.9910	75.2028
	0.5,1.5,1	0.9613	58.3007	0.9747	61.7893	0.9565	84.2533	0.9884	84.5588
	0.1,0.5,0.75	0.9587	22.2119	0.9750	28.0222	0.9564	24.0730	0.9858	26.9834
	0.5,1,0.25	0.9574	28.6244	0.9728	26.9279	0.9543	39.8667	0.9809	39.6730
20,20,20	0.1,0.1,0.1	0.9594	5.7031	0.9712	6.0774	0.9580	5.9319	0.9691	6.1473
	0.1,0.5,0.1	0.9582	8.6169	0.9691	8.7532	0.9554	13.7207	0.9638	13.5806
	1,0.75,0.5	0.9577	22.9289	0.9724	24.6418	0.9571	24.0631	0.9872	25.9005
	0.75,1,1.5	0.9543	50.9910	0.9728	72.8227	0.9574	47.9971	0.9844	61.5845
	0.5,1.5,1	0.9546	39.2394	0.9709	46.7208	0.9576	51.7822	0.9863	54.6033
	0.1,0.5,0.75	0.9500	20.8667	0.9737	27.2530	0.9532	20.7679	0.9828	24.2409
	0.5,1,0.25	0.9587	17.7673	0.9713	17.9988	0.9507	27.0370	0.9685	26.7414
20,30,25	0.1,0.1,0.1	0.9559	5.0226	0.9670	5.3080	0.9553	4.9201	0.9671	5.1153
	0.1,0.5,0.1	0.9552	6.8902	0.9660	7.0501	0.9473	10.3947	0.9566	10.3732
	1,0.75,0.5	0.9575	20.0288	0.9725	21.2970	0.9564	18.3640	0.9866	20.3904
	0.75,1,1.5	0.9537	37.1945	0.9739	51.2719	0.9500	33.2423	0.9848	42.4786
	0.5,1.5,1	0.9568	27.4858	0.9734	33.2876	0.9542	33.4687	0.9815	36.1806
	0.1,0.5,0.75	0.9509	16.7130	0.9728	21.3679	0.9527	15.9565	0.9805	18.6096
	0.5,1,0.25	0.9557	13.6648	0.9676	13.9695	0.9525	18.8581	0.9684	18.9923
20,50,30	0.1,0.1,0.1	0.9533	4.5523	0.9658	4.7823	0.9555	4.0650	0.9675	4.2502
	0.1,0.5,0.1	0.9542	5.6549	0.9624	5.8231	0.9515	7.7297	0.9590	7.7800
	1,0.75,0.5	0.9542	18.0376	0.9723	18.9668	0.9566	14.5129	0.9864	16.7826
	0.75,1,1.5	0.9564	30.5536	0.9777	41.1067	0.9557	25.3435	0.9859	32.8772
	0.5,1.5,1	0.9535	21.4944	0.9708	26.2101	0.9537	23.1301	0.9814	25.8298
	0.1,0.5,0.75	0.9531	14.2383	0.9739	17.9733	0.9525	12.8211	0.9798	15.1347
	0.5,1,0.25	0.9557	11.2700	0.9673	11.5982	0.9550	13.3872	0.9695	13.7760
50,50,50	0.1,0.1,0.1	0.9506	3.3833	0.9595	3.5202	0.9541	3.5151	0.9623	3.5954
	0.1,0.5,0.1	0.9513	4.7300	0.9581	4.8048	0.9518	7.4387	0.9580	7.4278
	1,0.75,0.5	0.9505	10.7206	0.9667	11.4515	0.9536	11.5016	0.9744	12.0732
	0.75,1,1.5	0.9529	19.1421	0.9736	24.3132	0.9489	18.4865	0.9792	21.5422
	0.5,1.5,1	0.9522	15.7517	0.9727	18.0950	0.9536	19.9353	0.9766	20.8584
	0.1,0.5,0.75	0.9543	10.3003	0.9767	12.2204	0.9519	10.3072	0.9781	11.3492
	0.5,1,0.25	0.9525	8.4831	0.9610	8.6867	0.9506	12.5104	0.9587	12.5322

Table 2: The estimated coverage probability and average length for $k = 5$ populations.

$n_1, \ldots, n_5$	$\sigma_1^2, \ldots, \sigma_5^2$	$c_1 = (1,1,-1.5,1,-1.5)'$				$c_2 = (-2,1.5,1,-1.5,1)'$			
		GCI		MOVER		GCI		MOVER	
		CP	AL	CP	AL	CP	AL	CP	AL
5,5,5,5,5	0.1,0.1,0.1,0.1,0.1	0.9924	33.1370	0.9956	33.6437	0.9905	38.9011	0.9949	39.3721
	0.1,0.5,0.1,0.5,0.1	0.9813	9792.783	0.9936	4030.561	0.9880	3876.62	0.9968	5128.42
	1,0.75,0.5,0.75,0.5	0.9913	49828806	0.9962	17112931	0.9946	5.5770e9	0.9972	9.8784e9
	0.75,1,1.5,1,1.5	0.9951	4.7656e21	0.9969	3.9151e21	0.9929	7.8542e16	0.9965	8.9771e15
	0.5,1.5,1,1.5,1	0.9932	1.8579e15	0.9966	6.0076e11	0.9912	5.3500e13	0.9951	1.2409e13
	0.1,0.5,0.25,0.5,0.25	0.9902	2718.32	0.9956	1367.38	0.9873	18903.89	0.9942	13543.42
	0.5,0.1,0.25,0.1,0.25	0.9903	4372.361	0.9959	5790.532	0.9890	1605.298	0.9933	2601.315
10,8,10,8,10	0.1,0.1,0.1,0.1,0.1	0.9726	11.5303	0.9835	12.2612	0.9754	14.1143	0.9876	15.3354
	0.1,0.5,0.1,0.5,0.1	0.9648	40.8378	0.9815	35.7483	0.9732	58.0039	0.9899	69.6498
	1,0.75,0.5,0.75,0.5	0.9758	155.9292	0.9853	125.0432	0.9835	224.8003	0.9946	280.5307
	0.75,1,1.5,1,1.5	0.9876	909.3059	0.9937	1147.0038	0.9849	854.3197	0.9894	786.9367
	0.5,1.5,1,1.5,1	0.9840	8529.296	0.9901	6738.908	0.9841	5966.566	0.9907	6913.764
	0.1,0.5,0.25,0.5,0.25	0.9754	45.8658	0.9876	42.6040	0.9762	59.4592	0.9891	69.8265
	0.5,0.1,0.25,0.1,0.25	0.9738	25.0703	0.9867	27.3969	0.9706	34.4134	0.9853	45.4091
10,15,10,15,10	0.1,0.1,0.1,0.1,0.1	0.9703	9.9281	0.9829	10.9900	0.9697	11.2239	0.9804	12.2084
	0.1,0.5,0.1,0.5,0.1	0.9701	16.2606	0.9789	16.4381	0.9708	22.2001	0.9858	25.1124
	1,0.75,0.5,0.75,0.5	0.9782	75.6315	0.9877	78.9525	0.9742	116.5807	0.9894	171.5774
	0.75,1,1.5,1,1.5	0.9812	636.2624	0.9955	961.4533	0.9771	435.3768	0.9843	370.8010
	0.5,1.5,1,1.5,1	0.9847	179.2753	0.9929	228.3742	0.9799	175.8385	0.9864	176.9196
	0.1,0.5,0.25,0.5,0.25	0.9750	22.2611	0.9860	25.1736	0.9723	24.6805	0.9836	26.7827
	0.5,0.1,0.25,0.1,0.25	0.9733	24.0208	0.9894	26.7664	0.9656	32.6531	0.9807	44.0068
10,15,20,15,10	0.1,0.1,0.1,0.1,0.1	0.9640	8.7386	0.9785	9.4660	0.9680	10.7377	0.9807	11.8045
	0.1,0.5,0.1,0.5,0.1	0.9632	15.4401	0.9730	15.3521	0.9641	21.9007	0.9842	24.8741
	1,0.75,0.5,0.75,0.5	0.9732	68.3942	0.9844	68.8458	0.9764	110.2597	0.9905	165.5575
	0.75,1,1.5,1,1.5	0.9815	314.8482	0.9937	519.6759	0.9782	265.1290	0.9841	266.9136
	0.5,1.5,1,1.5,1	0.9796	125.8911	0.9887	153.3913	0.9804	144.2265	0.9888	160.4106
	0.1,0.5,0.25,0.5,0.25	0.9704	19.4878	0.9824	21.3551	0.9687	23.6206	0.9831	26.1823
	0.5,0.1,0.25,0.1,0.25	0.9688	21.2137	0.9853	22.8988	0.9635	31.5553	0.9807	43.5690
20,20,20,20,20	0.1,0.1,0.1,0.1,0.1	0.9605	6.4030	0.9721	6.8023	0.9609	7.5712	0.9734	8.0362
	0.1,0.5,0.1,0.5,0.1	0.9566	11.3421	0.9662	11.3315	0.9583	15.9905	0.9811	17.8410
	1,0.75,0.5,0.75,0.5	0.9672	26.4016	0.9782	27.9785	0.9702	37.8923	0.9890	48.0635
	0.75,1,1.5,1,1.5	0.9688	62.1463	0.9889	85.1021	0.9702	57.1383	0.9801	60.7679
	0.5,1.5,1,1.5,1	0.9713	52.2671	0.9829	58.2421	0.9694	63.0132	0.9834	73.9438
	0.1,0.5,0.25,0.5,0.25	0.9626	13.6264	0.9783	14.4691	0.9645	16.7325	0.9810	18.4960
	0.5,0.1,0.25,0.1,0.25	0.9635	11.7699	0.9816	12.8827	0.9593	15.6546	0.9774	18.9791
20,30,25,30,20	0.1,0.1,0.1,0.1,0.1	0.9588	5.8079	0.9705	6.1746	0.9580	6.8108	0.9694	7.2331
	0.1,0.5,0.1,0.5,0.1	0.9589	8.8634	0.9676	9.0239	0.9585	12.2035	0.9777	13.3691
	1,0.75,0.5,0.75,0.5	0.9663	22.6567	0.9796	24.5140	0.9630	32.8799	0.9825	42.9061
	0.75,1,1.5,1,1.5	0.9705	50.6831	0.9916	70.7330	0.9652	45.5158	0.9780	49.0005
	0.5,1.5,1,1.5,1	0.9726	36.7724	0.9851	44.1543	0.9701	40.7960	0.9821	46.3060
	0.1,0.5,0.25,0.5,0.25	0.9602	11.1553	0.9771	12.1131	0.9558	13.0022	0.9744	14.0742
	0.5,0.1,0.25,0.1,0.25	0.9603	11.0975	0.9785	12.0552	0.9541	15.1114	0.9760	18.4910
20,50,30,50,30	0.1,0.1,0.1,0.1,0.1	0.9533	5.3838	0.9681	5.7360	0.9539	6.2391	0.9678	6.6389
	0.1,0.5,0.1,0.5,0.1	0.9599	7.1845	0.9688	7.4305	0.9578	9.4694	0.9761	10.2130
	1,0.75,0.5,0.75,0.5	0.9649	20.7940	0.9797	22.6825	0.9573	30.0954	0.9799	40.1817
	0.75,1,1.5,1,1.5	0.9644	45.0786	0.9882	63.9950	0.9653	39.8508	0.9762	43.3396
	0.5,1.5,1,1.5,1	0.9681	29.9263	0.9860	38.0371	0.9648	30.1800	0.9774	33.5374
	0.1,0.5,0.25,0.5,0.25	0.9543	9.5875	0.9741	10.6214	0.9569	10.3958	0.9717	11.0521
	0.5,0.1,0.25,0.1,0.25	0.9573	10.6593	0.9766	11.5221	0.9544	14.7059	0.9754	18.1006
50,50,50,50,50	0.1,0.1,0.1,0.1,0.1	0.9569	3.7778	0.9644	3.9211	0.9495	4.4701	0.9586	4.6378
	0.1,0.5,0.1,0.5,0.1	0.9552	6.0372	0.9599	6.0952	0.9558	8.3824	0.9725	8.9930
	1,0.75,0.5,0.75,0.5	0.9541	12.0434	0.9655	12.8099	0.9573	16.8420	0.9787	19.7809
	0.75,1,1.5,1,1.5	0.9612	21.7064	0.9854	27.2191	0.9532	20.9476	0.9713	22.8506
	0.5,1.5,1,1.5,1	0.9618	18.9136	0.9772	21.1688	0.9583	22.5071	0.9773	25.4275
	0.1,0.5,0.25,0.5,0.25	0.9565	7.2240	0.9671	7.5691	0.9569	8.7871	0.9723	9.3741
	0.5,0.1,0.25,0.1,0.25	0.9531	6.4119	0.9676	6.8122	0.9499	8.3754	0.9718	9.4225

4 Simulation study

In this section, through simulations, we compare the performances of the proposed generalized confidence interval with the method of variance estimate recovery method in terms of the coverage probability (CP) and the average length (AL). We considered different values and scenarios for $\boldsymbol{\sigma^2} = (\sigma_1^2, \cdots, \sigma_k^2)$ and $\mathbf{n} = (n_1, \cdots, n_k)$. Simulation is done for $\mu_i = 1$ for $i = 1, \cdots, k$ and different values of $\boldsymbol{\sigma^2}$ and $\mathbf{n}$ at level 95%. Note that the estimated CPs are approximately normal variables with the mean 0.95 and the variance 0.95×0.05/10000. Therefore, a 95 lower bound for estimated CPs is 0.946. If the estimated CP of a method is less than 0.946, we can conclude that the method is liberal. We note that a method is preferred to other methods when its estimated CP is not significantly less than 0.95 and it has the smallest AL among the methods that are not liberal.

(1) For different values of $\boldsymbol{\sigma^2}$ and $\mathbf{n}$, all of CPs of two approaches are not liberal, i.e. are not significantly less than 95%.

(2) In small and moderate sample sizes, the average lengths of the GCI and MOVER methods are not well comparable. In large sample sizes, it seems that the GCI results shorter confidence interval than MOVER method.

(3) **Final simulation conclusion.** Our simulation results show that the two methods give reliable confidence intervals for the different values of sample sizes and variances that we considered here. In addition, the MOVER approach is simpler than GCIs to implement since it does not need computer simulation. Therefore, the MOVER is recommended for small and moderate sample sizes. Also, the GCI is recommended for large sample sizes.

5 Numerical examples

Based on the medical charge data, the sample mean (sample variance) of the log-transformed data for the African-American group and the white group are 9.067 (1.824) and 8.693 (2.693), respectively. The 95 two-sided GCI, and MOVER confidence intervals for the different linear combination are shown in Table 3. We also presented the lengths of confidence intervals in this table.

Zhou et al. (1997) and Wu et al. (2002) shown that is no significant differences between the means of this data, i.e. $\eta_1^* = \eta_2^*$. It can be seen from the results of table 3 that both confidence intervals for coefficient 1 and -1 contain zero value.

Table 3. The 95% confidence intervals for the different linear combination mean charges.

c_1, c_2	Methods	Confidence interval	Length
1, -1	GCI	(-19402.84, 11391.53)	30794.37
	MOVER	(-15467.49, 14027.53)	29495.03
-1, 2	GCI	(5347.403, 58774.75)	53427.35
	MOVER	(8055.457, 51815.57)	43760.11
3, -2	GCI	(-18932.17, 52497.58)	71429.75
	MOVER	(-11506.28, 54306.54)	65812.82

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