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Multi-objective optimal design strategy under type-II progressive censoring with random dependent removal model

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Abstract: In designing an optimal life-testing experiment under a censoring setup, the removal vector scheme is usually chosen by optimizing a suitable criterion function. The criterion functions are usually constructed based on cost or variance functions, and sometimes a combination of both. This paper considers a multiple optimization problem in the context of type-II progressive censoring with random dependent removal lifetime experiment. A simple simulation algorithm is presented for obtaining the optimal scheme in a multi-objective optimal problem under the type-II progressive censoring with random dependent removal model. Several simulation studies are conducted to evaluate and compare the performance of the proposed strategy. Finally, some concluding remarks and future works are provided.

Keywords: Cost function; Dependent random removal mechanism; Multi-objective optimal design.
Mathematics Subject Classification (2010): 62N01, 62J12.

1 Introduction

The single-objective optimal designs have been used in selecting an efficient design in the lifetime experiment. Single-objective optimal designs might be criticized for not covering all aspects of the experiment. When the experimenter has multiple goals, especially more than two, these designs must be revised. Rising different aspects of an experiment have necessitated further efficient use of novel approaches. When two

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or more competing criteria are combined, considering multi-objective optimal (MOO) design for obtaining efficient design is reasonable. A MOO design is more valuable than a single-objective optimal design because most experiments have several objectives in practice. Actually, MOO designs are a popular topic for considering all aspects of an experiment with different methods. For elaborate discussion and profound description of the different MOO design and their related issues, the reader is encouraged to study the books by Ehrgott (2005). The constrained and compound optimal strategies are two standard methods in the MOO problem, which had been investigated by Huang and Wong (1998) and Huang (1996).

In the lifetime experiments, mostly, single-objective optimal designs have been considered. Since the lifetime experiments are usually conducted under some censoring mechanism, finding the optimal censoring scheme is desired using a suitable optimality criterion. It can be done by maximizing or minimizing a proper choice of criterion function based primarily on some variance or cost measures. Many authors have studied the single-objective optimal design problem, such as; Balakrishnan and Aggarwala (2000) were the first to discuss an optimal progressive censoring scheme considering the variance of the best linear unbiased estimators of the model parameters as the optimality criterion under normal and extreme value distributions. Burkschat et al. (2006, 2007) computed optimal censoring schemes for generalized Pareto distribution using the same optimality criterion. Moreover, Ng et al. (2004) considered minimum variance criteria for the maximum likelihood estimator of the Weibull distribution. Based on the Fisher information matrix, they introduced three optimality criteria. Fisher information matrix as optimality criteria has also been discussed by Abo-Eleneen (2007), Balakrishnan et al. (2008), Cramer and Ensenbach (2011), and Cramer and Schmiedt (2011). Pradhan and Kundu (2009, 2013) proposed an optimality criterion based on an asymptotic variance of the estimator of the p^{th} quantile of underlying lifetime distribution. They obtained the optimum schemes for generalized exponential and Birnbaum-Saunders distributions. The problem of optimum type-II progressive censoring scheme via a meta-heuristic algorithm based on a variable neighborhood search approach has been considered by Bhattacharya et al. (2016). They showed that the variable neighborhood search algorithm performs sufficiently well for moderate to large sample size values. Recently, Balakrishnan and Bhattacharya (2021) presented a simple probabilistic approach for determining an optimal progressive censoring scheme. They considered both variance measure and cost measure as optimality criteria.

Recently, the MOO designs have received more attention in planning lifetime experiments. Bhattacharya et al. (2019) presented the constrained optimization problem for censoring life-testing design. They considered multi-criteria for optimization problems under a hybrid censored scheme. Actually, they proposed an algorithm to compute optimal design by converting the multi-criteria setup to max-min setup. Also, Bhattacharya (2020) introduced the concept of compound optimal design to find an optimal scheme in type-II progressive censoring. This strategy is constructed based on two convex functions, and the optimal designs are a trade-off between two considered criteria. Hassantabar et al. (2023) recently presented a new optimal design approach for optimizing two criteria.

The present article develops a MOO strategy in which an experiment's desired goals are simultaneously optimized. A simple algorithm is also proposed to compute the so-determined optimal design. The solution strategy is described based on two introduced criteria in a life-testing under type-II progressive censoring with random dependent removal, which will be denoted by the "type-II PCRD" hereafter. The type-II PCRD model was introduced by Hassantabar et al. (2022). In the type-II PCRD model, the number of dropouts at each failure time follows a conditional binomial distribution with dependent success probabilities. The success probabilities are updated based on failure distances at each stage by adjusting some tuning parameters. The tuning parameters lead to more flexibility in removal patterns, so it is essential to determine their optimal value in some applications according to suitable criteria. In subsequent sections, we present the general strategy of MOO design and the criteria used to optimize simultaneously.

The rest of the paper is organized as follows. In Section 2, the problem of the MOO designs is presented. Also, some criteria which are used as optimality criteria are introduced. type-II PCRD has been reviewed in Section 3. Then, the strategy of MOO design is presented under type-II PCRD for the Weibull distribution. In Section 4, several simulation studies are performed. Finally, Section 5 contains some concluding remarks and comments.

2 The multi-objective optimal design strategy

This section describes an approach for constructing optimal life-testing plan in multi criteria setup. First, we formulate an appropriate criterion in the context of type-II PCRD model and then present the method for achieving the solution. In type-II progressive censoring, choosing and removing an optimal number of unfailed units as censoring, may be a serious challenge which has led to a lot of researches in this area. The progressive censoring experiment starts with n units and terminates after observing the m^{th} failure. Upon a failure, a predetermined number of units is removed from the test. This procedure continues till the m^{th} failure is observed and then, all remaining units are withdrawn. If R_i denotes the number of removed units after the i^{th} failure, the censoring scheme is referred to as $\boldsymbol{\mathcal{R}} = (R_1, \ldots, R_m)$ where R_i's are non-negative integer numbers where $R_1 + \cdots + R_m = n - m$. In practice, m and R_i's are fixed before the experiment starts. Let $CS(n, m)$ denote the set of all admissible censoring schemes for n units and m failures where $m \leq n$ and let us denote those as,

$$CS(n,m) = \{\boldsymbol{\mathcal{R}} = (R_1, \cdots, \mathcal{R}_m) \in \mathbb{N}^m | \sum_{i=1}^{m} \mathcal{R}_i = n - m, \mathbb{N} = \{0, 1, 2, \ldots, n - m\}\}.$$

In designing a life-testing, the main problem is which element of $CS(n, m)$ should be used in order to maximize the accuracy and minimize the cost and time. Such an element of $CS(n, m)$ is called an optimal design. A high experimental cost directly affects the budget constraint of the manufacturer, whereas a poor estimation of underlying model parameters may incur heavy post-sale expenses. Therefore optimal designs are selected based on desired optimization criteria. We adapted a simple approach for constructing multi-objective locally optimal designs for a typical type-II progressive censoring setup. For this purpose, two single-objective criteria based on cost functions have been considered as follows.

Criterion I: The censoring cost of an experiment as a single-objective criterion function is defined. In type-II progressive censoring, experiment units are censored upon a failure which imposes a side cost. We suppose that the cost of censoring in different stages is not the same, and censoring in higher stages is more costly than in the lower stages. The components of censoring cost are considered as: the repair and reinstall cost, C_{0r}, the cost per failed item, C_m, and vector of censoring cost for random removal vector, $\mathcal{C}_r$, which is assumed to be larger in higher stages. The total expected censoring cost for removal vector $\boldsymbol{\mathcal{R}}$ is given by,

$$\phi_1(\boldsymbol{\mathcal{R}}) = C_{0r} + mC_m + E(\mathcal{C}_r'\boldsymbol{\mathcal{R}}). \tag{1}$$

Criterion II: The second single-objective criterion is developed based on the duration of the experiment. The total expected experiment time, which is widely noticed in the literature, is considered as,

$$\phi_2(\boldsymbol{\mathcal{R}}) = C_{0t} + (n-m)C_n + C_tE(T_{m:m:n}), \tag{2}$$

where, C_{0t}, C_n and C_t are installation cost or transportation cost of testing facilities, sample cost, which is the cost of running the experiment with sample size n and cost of the duration of the experiment, respectively. Here $C_{0r}, C_m, \mathcal{C}_r$, C_{0t}, C_n and C_t, are determined by an experimenter. These introduced single-objective criteria are based on cost functions and are used to construct MOO designs strategy in which all the desired goals of an experiment are simultaneously optimized. Consequently, the optimal design is obtained by minimizing all criteria simultaneously.

Suppose $\phi_1(\cdot), \ldots, \phi_k(\cdot)$ are k optimality criteria over the design space $\xi \in \chi$. The general MOO problem is posed as follows,

$$minimize : \phi_{12\cdots k}(\xi) = (\phi_1(\xi), \phi_2(\xi), \cdots, \phi_k(\xi)). \tag{3}$$

The idea of a solution for (3) can be unclear, because a single point that minimizes all objectives usually does not exist. One of the most intuitive methods for solving a MOO is optimizing a weighted sum of the objective functions using any method for single objective optimization. The MOO is transformed into a single objective problem by using a weighted sum of the original multiple objectives as follows,

$$minimize : \phi(\xi|w_i, i = 1, \ldots, k-1) = \sum_{i=1}^{k-1} w_i\phi_i(\xi) + (1 - \sum_{i=1}^{k-1} w_i)\phi_k(\xi), \tag{4}$$

where w_i's are the weighting coefficients. There are different method for determining the weighting coefficients and in some cases will only determine based on the decision makers preferences. The method is simple to implement but the results obtained are highly dependent on the weights used, which have to be specified before the optimization process begins. This method is simple to implement, but the results highly depend on the weights. Additionally, in the presence of the convex functions, a complete set of Pareto solutions can be obtained by varying the weighting coefficients. The following section presents our strategy for determining the weighting coefficient under the type-II PCRD model.

3 General multi-objective optimal strategy

This section use MOO design approach for optimization the type-II PCRD model and, describe a general strategy to solve the optimization problem in (4). Assuming, $X_1, \ldots, X_n$ are independent and identically distributed random variables from an absolutely continuous population with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$. Let $T_{1:m:n}, \ldots, T_{m:m:n}$ denote the corresponding the type-II progressive censoring order statistics under a random removal scheme $\boldsymbol{\mathcal{R}} = (R_1, \ldots, R_m)$ where $R_i, i = 1, \ldots, m$, are random variables and belong to $[0, n-m-(R_1+R_2+\cdots+R_{i-1})]$. Based on the GLM-based dependent random removal mechanism proposed by Hassantabar et al. (2022), R_is have some conditional binomial distribution with success probabilities dependent on the experimental conditions. In this work, these conditions, at the i^{th} stage, have been considered preceding number of removals $(R_1, \ldots, R_{i-1})$ and observed failure times $(T_{1:m:n}, \ldots, T_{i:m:n})$. That is,

$$\begin{aligned} R_1 \quad &| \quad T_{1:m:n} \sim b(n-m, p_1), \\ R_i \quad &| \quad R_1, \ldots, R_{i-1}, T_{1:m:n}, \ldots, T_{i:m:n} \sim b(n-m-\sum_{j=1}^{i-1} R_j, p_i), \quad i = 2, \ldots, m-1, \\ R_m \quad &= \quad n - \sum_{i=1}^{m-1} R_i - m, \end{aligned}$$

where, p_is, binomial parameters, are modeled as follows,

$$H(p_i) = \alpha_0 + \alpha_1(T_{i:m:n} - T_{i-1:m:n}), \quad i = 1, \ldots, m-1,$$

where, $T_{0:m:n} = 0$ and $\{\alpha_0, \alpha_1\}$ are the set of tuning parameters leading to different removal probabilities according to the goals of study. As a special case, *logit* link function (i.e., $logit(p_i) = log(\frac{p_i}{1-p_i})$) is used to model $H(\cdot)$ function. Let $\underline{\mathbf{t}} = (t_{1:m:n}, \cdots, t_{m:m:n})$, and $\underline{\mathbf{r}} = (r_1, \cdots, r_m)$ denote the observed failure time and the vector of observed number of removals. The likelihood function for the observed sample under the type-II PCRD is represented as,

$$\begin{aligned} L(\Theta; \underline{\mathbf{r}}, \underline{\mathbf{t}}) \quad = \quad & C_R f(t_1)[1-F(t_1)]^{n-1} \Pi_{i=2}^{m} \frac{f(t_i)}{1-F(t_{i-1})} \left(\frac{1-F(t_i)}{1-F(t_{i-1})} \right)^{n-\sum_{j=1}^{i-1} r_j - i} \\ & \times \Pi_{i=1}^{m-1} \binom{n-m-\sum_{j=1}^{i-1} r_j}{r_i} (\exp(\alpha_0 + \alpha_1(t_{i:m:n} - t_{i-1:m:n})))^{r_i} \\ & \times (1 + \exp(\alpha_0 + \alpha_1(t_{i:m:n} - t_{i-1:m:n})))^{-(n-m-\sum_{j=1}^{i-1} r_j)}, \end{aligned}$$

where, $C_R = n \prod_{i=2}^{m}(n - \sum_{j=1}^{i-1} r_j - i + 1)$ and Θ is the vector of parameters of distribution. The Weibull lifetime distribution is considered for $F(\cdot)$ with cdf $F(x; \gamma, \beta) = 1 - e^{-(\frac{x}{\gamma})^{\beta}}$.

For solving the MOO under the type-II PCRD in (4), let us first consider censoring cost and duration of experiment cost as two single-objective criteria in the type-II PCRD model. Therefore, based on (1) and (2), we have,

$$\phi_1(\boldsymbol{\mathcal{R}}) \quad = \quad C_{0r} + mC_m + E[\mathcal{C}_r \boldsymbol{\mathcal{R}} | \mathcal{P}(\alpha_0, \alpha_1)],$$

$$\phi_2(\boldsymbol{\mathcal{R}}) \quad = \quad C_{0t} + nC_n + C_t E[T_{m:m:n}|\mathcal{P}(\alpha_0, \alpha_1)],$$

where, $\mathcal{P}(\alpha_0, \alpha_1) = (p_1, \ldots, p_{m-1})$. Note that, the optimal removal vector ($\boldsymbol{\mathcal{R}}^*$) is specified with optimal tuning parameters ($\alpha_i^*, i = 0, 1$). Therefore, the optimal removal vector in the type-II PCRD model is determined by finding the optimal value of tuning parameters. The steps of MOO design strategy under the type-II PCRD is as follows,

1. Let us normalize optimal criteria, $\phi_i(\cdot), i = 1, 2$, and define the new objective functions as follows,

$$\Psi_i(\boldsymbol{\mathcal{R}}) = \frac{\phi_i(\boldsymbol{\mathcal{R}}) - mean(\phi_i(\boldsymbol{\mathcal{R}}))}{var(\phi_i(\boldsymbol{\mathcal{R}}))}, i = 1, 2, \tag{5}$$

   where, *mean* and *var* are the mean and variance of criteria for all possible designs (here, all possible removal vector). One of the appealing properties of normalized measure is that its interpretation remains the same regardless of the measurement unit and size of the design space. Since the considered criteria here are different in the measurement unit, this index is handy for combining and comparing criteria.

2. The coefficients of the equation are determined according to the range of changes of the criteria. In this way, by placing the range of changes of the smaller function in the numerator and the range of changes of the larger function in the denominator, we determine the coefficient for the larger function and place its complement for the other function.

$$w_i = \frac{\Psi_j^{max}(\boldsymbol{\mathcal{R}}) - \Psi_j^{min}(\boldsymbol{\mathcal{R}})}{\Psi_i^{max}(\boldsymbol{\mathcal{R}}) - \Psi_i^{min}(\boldsymbol{\mathcal{R}})}. \tag{6}$$

3. For $w_i \in [0, 1]$, the design which minimizes Ψ_{12} is chosen and called optimal design and is denoted by $\boldsymbol{\mathcal{R}}^*$ and its corresponding tuning parameters in the GLM-based mechanism, $\alpha_i^*, i = 0, 1$.

$$\Psi_{ij} = w_i \Psi_i(\boldsymbol{\mathcal{R}}) + (1 - w_i)\Psi_j(\boldsymbol{\mathcal{R}}). \tag{7}$$

It is worth to note, the optimal design, $\boldsymbol{\mathcal{R}}^*$, of $\phi_{12}(\cdot)$ is neither an optimal design for $\phi_1(\cdot)$ nor $\phi_2(\cdot)$ individually but is optimal simultaneously for both of them.

4 Simulation study

Here, we used the proposed MOO design strategy to obtain optimal design in presence of the type-II PCRD samples. The simulation study has been conducted to generate 10,000 the type-II PCRD samples from the Weibull distribution with the shape parameters $\beta \in \{0.5, 2\}$ and scale parameter of $\gamma = 6$. we set $(C_{0t}, C_n, C_t) = (100, 2, 12)$, $(C_{0r}, C_m, \mathcal{C}_r) = (50, 4, 8(1 : m))$ and $(n, m) = (6, 3)$. Also, according to proposed tuning parameters value by Hassantabar et al. (2022), we set $\alpha_0 = 0.5$ and optimize α_1 value. The simulation algorithm for obtaining the optimal design under the type-II PCRD model is as follows.

Generate 10,000 type-II PCRD samples from $F(\cdot)$ distribution with dependent removal scheme $\boldsymbol{\mathcal{R}}$.
Compute $\phi_1(\cdot)$ and $\phi_2(\cdot)$ base on $\mathcal{T}$ and $\boldsymbol{\mathcal{R}}$.
Obtain $\Psi_1(\cdot)$ and $\Psi_2(\cdot)$ based on (5).
Determine w_i by (6).
Minimize the compound design of $\Psi_{12}(\cdot)$, (7).

In Figure 1 and 2, $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are drawn versus values of α_1. As shown in this figure, the single-objective criteria have opposite trends, and the optimal design of every single-objective function, $\phi_1(\cdot)$ and $\phi_1(\cdot)$, would be obtained through numerical methods assuming large and small values of the tuning parameters, respectively. Since the range of changes of the two criteria is different, the two criteria are normalized. Then, by determining the weighting coefficient and minimizing (7), the optimal tuning parameters for $\beta = 0.5$ and $\beta = 2$ are obtained and shown in Figure 1 and 2, respectively.

A comparative study between the single-objective and MOO designs has been conducted for further investigation. For each optimal design, the expected experiment time, expected censoring cost, and variance of the logarithm of p^{th} quantile of the lifetime distribution is reported in Table 1. The expected experiment time is computed based on the sample average of the m^{th} order statistics. The cost of censoring in different stages is not the same; therefore, the average cost of the random removal vector is computed for the expected censoring cost. The variability of the quantile estimator of a particular censoring scheme is measured by the variance of the logarithm of p^{th} quantile. See Hassantabar et al. (2022) for more information about these indicators. Table 1 shows that the shortest expected censoring cost occurs when the expected experiment time is the highest under the $\phi_1(\cdot)$ criterion. However, minimization of an experiment time under $\phi_2(\cdot)$ results in higher expected censoring and variance of the logarithm of p^{th} quantile cost. Therefore, the MOO design balances both effects of $\phi_1(\cdot)$ and $\phi_2(\cdot)$ by keeping their values between the individual optimal criteria values. Table 1 shows that the MOO design is a reasonable approach to the trade-off between two criteria.

Table 1: Comparing between the single-objective optimal designs and the MOO design.

β	design	Criterion	$E(T_m)$	$E(C_r'\boldsymbol{\mathcal{R}})$	$V(ln(T_p))$	α_1^*	R_i^*
0.5	single-objective	ϕ_1	17.00627	36.04000	1.34379	4.99	$R_1^* =$(2, 1, 0)
		ϕ_2	4.70811	58.74400	1.36928	-4.96	$R_2^* =$(1, 0, 2)
	MOO	ϕ_{12}	10.50779	45.27200	1.35176	0.38	$R_{12}^* =$(1, 1, 1)
2	single-objective	ϕ_1	7.217261	24.296	0.415583	3.86	$R_1^* =$(3, 0, 0)
		ϕ_2	4.534232	70.568	0.514023	-5	$R_2^* =$(0, 0, 3)
	MOO	ϕ_{12}	5.319304	52.504	0.470471	-0.16	$R_{12}^* =$(1, 1, 1)

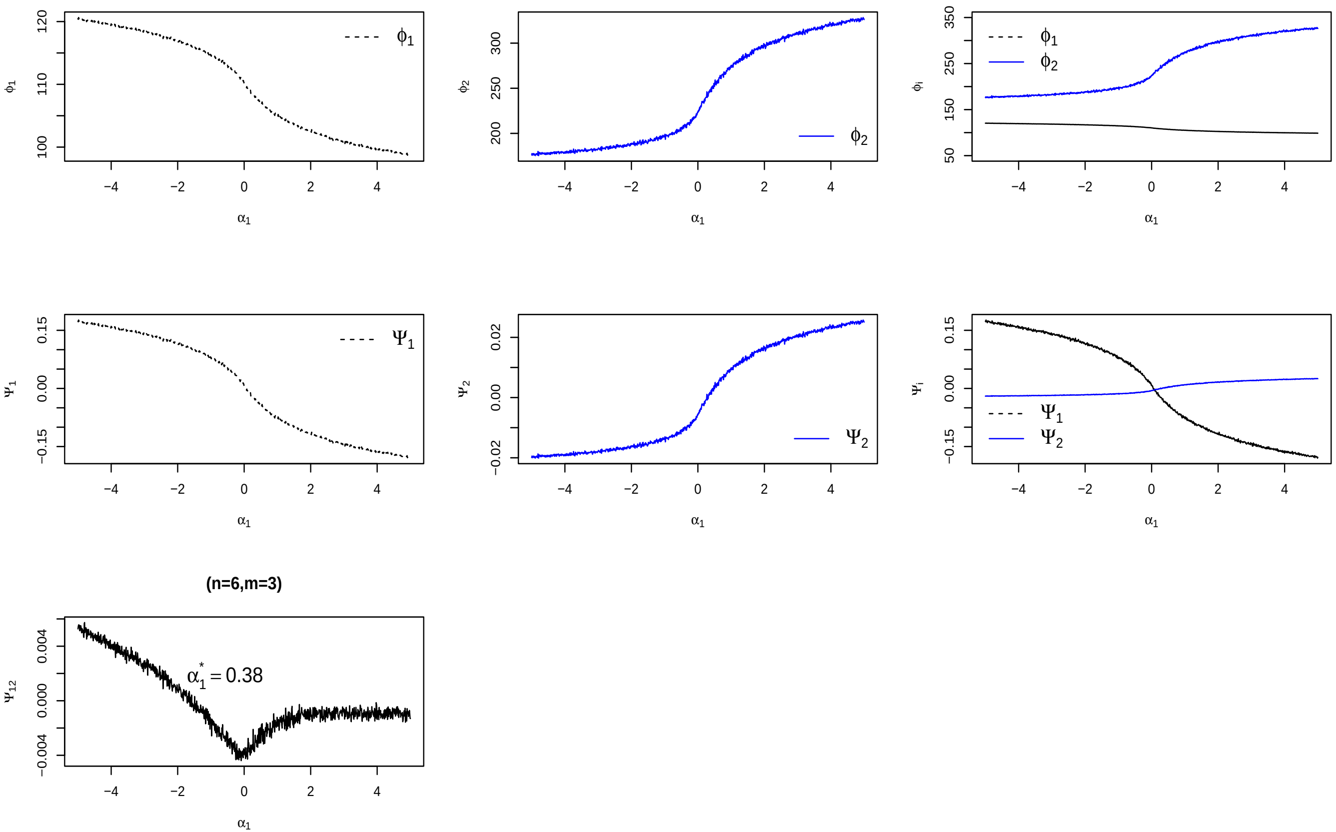

Figure 1: Plot of two criteria , ϕ_1, ϕ_2, its normalized criteria, Ψ_1, Ψ_2 and Ψ_{12} versus α_1 values assuming $(\beta, \gamma) = (0.5, 6)$ and $\alpha_0 = -0.5$ and $(n = 6, m = 3)$.

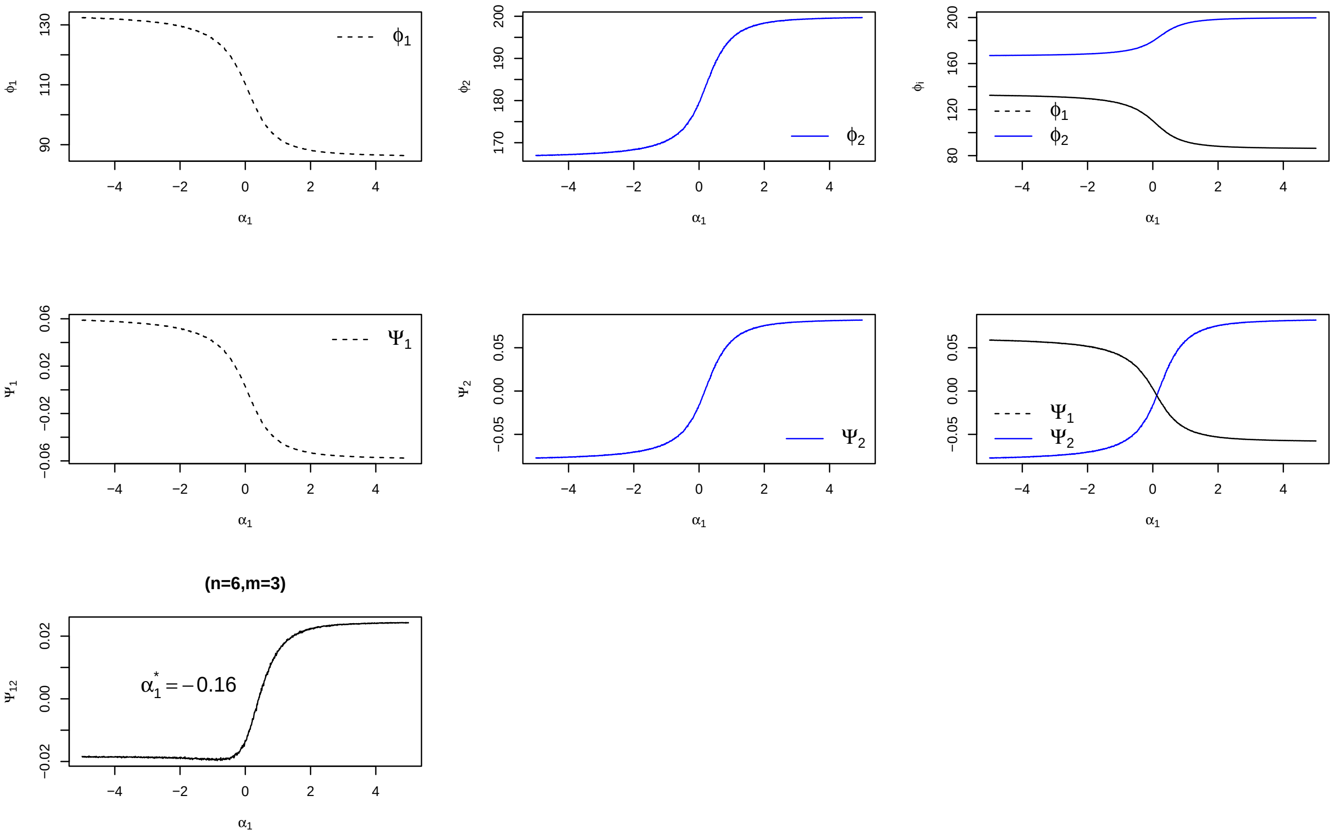

Figure 2: Plot of two criteria , ϕ_1, ϕ_2, its normalized criteria, Ψ_1, Ψ_2 and Ψ_{12} versus α_1 values assuming $(\beta, \gamma) = (2, 6)$ and $\alpha_0 = -0.5$ and $(n = 6, m = 3)$.

5 Conclusion and future research

This work describes implementing the MOO design strategy in censored life-testing experiments and provides a procedure for obtaining the solution. In particular, the type-II progressive censoring sample of lifetime data with a novel GLM-based random removal mechanism as the type-II PCRD model, where the number of dropouts at each failure time follows a conditional binomial distribution with dependent success probabilities, which is introduced by Hassantabar et al. (2022), is considered. This stochastic removal mechanism can update the removal probability at each stage by adjusting some tuning parameters. The tuning parameters lead to more flexibility in removal patterns, so it is important to determine their optimal value in some applications according to suitable criteria. The optimal tuning parameter values of the GLM-based mechanism and corresponding removal vector are obtained via the MOO methodology. The proposed solution methodology is computationally simple. In multi-criteria formulation, we use two objective functions. However, adding more suitable objective functions can extend this to more complex structures. We use the weighing and normalizing strategy in the MOO problem. Also, the Weibull lifetime distribution under the type-II PCRD model is used to determine the optimal removal vector. Theoretically, one can work with other optimization solutions and lifetime models by considering different criteria.

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