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Estimation of the stress-strength reliability for the Levy distribution based on the ranked set sampling

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Abstract: In this paper, the problem of inferencing the stress-strength reliability under the ranked set sampling and the simple random sampling from the levy distribution function is investigated. The maximum likelihood estimators, their asymptotic distributions, and Bayes estimators are provided for the stress-strength reliability parameter. Furthermore, using a Monte Carlo simulation, for both sampling methods, namely, simple random sampling and ranked set sampling, the Bayes risk estimators and the efficiency of the obtained estimators are computed and compared.

Keywords: Maximum likelihood estimator; Ranked set sampling; Stress-strength reliability.
Mathematics Subject Classification (2010): 62D05, 62N05.

1 Introduction

The Levy distribution is a specific case of the inverted gamma distribution with the shape parameter $\frac{1}{2}$ and scale parameter $\frac{2}{\sigma_x}$; (O'Reilly and Rueda , 1998). The Levy distribution is not always used, but it has been very effective in the analysis of stock prices. Montroll and Shlesinger (1983) and Jurlewicz and Weron (1993) have applied the Levy distribution in physics. Levy distribution has experienced applications in many applied areas, containing dispersive transport in disordered semiconductors, stock and stock-indexes returns, linear dynamical systems, income distribution, stochastic artificial neural networks, many-particle quantum systems, and oil pricing time-series.

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A random variable X is said to have the Levy distribution with probability density function (PDF)

$$f\left(x;\, \sigma_x\right) = \sqrt{\frac{\sigma_x}{2\pi}} x^{-\frac{3}{2}} \exp\left(-\frac{\sigma_x}{2x}\right), \tag{1}$$

and the cumulative distribution function (CDF)

$$F\left(x;\, \sigma_x\right) = 2 - 2\Phi\left(\sqrt{\frac{\sigma_x}{x}}\right),$$

where $\sigma_x > 0$ is a scale parameter and $\Phi\left(.\right)$ denoted the CDF of standard normal distribution. Due to the heavy tail and stable family of Levy distribution, there is no variance and the expected values associated with it. The stress-strength model has collected large consideration in the reliability background and an extensive range of research in the literature is existed about this model. The quantity $R = P(X < Y)$, which is termed the stress-strength reliability, is a measure of assurance of the component performance with the random strength Y when it is subjected to the random stress X. In this model, whenever the applied stress is greater than its strength, the system will be failed. A large number of existing studies in the broader literature are devoted to the parametric and non-parametric inferences on R. A more comprehensive description can be found in Kotz et al. (2003) and references mentioned therein that contain an extensive review of the topic up to 2003. Further development of this model has been driven by Kundu and Gupta (2005), Kundu and Gupta (2006), Eryilmaz (2008), Kundu and Raqab (2009), Rezaei et al. (2010), Eryilmaz (2011), Pakdaman and Ahmadi (2013), Pakdaman and Ahmadi (2018), Condino et al. (2018), Guo and Gui (2018), Kumar and Siju (2019) and Xavier and Jose (2021). The statistical inference of the stress-strength model is popularly investigated based on simple random sampling (SRS). In some practical problems, obtaining observations for the variable of interest is costly and time consuming. In such situations, considering appropriate sampling schemes, in order to reduce the cost and increase efficiency, are worthwhile. These properties motivated (McIntyre, 1952) to present a primary sampling method called ranked set sampling (RSS). The concept of RSS is a particularized method managed for conditions where the quantification of an item is costly and hard, but the items are comfortably collected and the set of given numbers can be sorted justly successfully.

An RSS sampling scheme is planned according to the idea of ranking the sampling units with regard to virtual comparisons, expert opinions, or auxiliary variables. To obtain a ranked set sample of size n, select m random sets via SRS each of size m and rank them from the smallest to the largest without doing any certain measurements. Then, determine the observations which are used for certain measurements. To do this, select the smallest observation in the first set. In the second set, select the second smallest observation. Continue this process until the last set. In the last set, select the largest observation for a certain measurement. This whole process is called a cycle. By repeating this process r time, the sample size is obtained as $n = mr$. Here, m and r refer to the set size and number of cycles, respectively. Over the last few years, the RSS method is used for the construction of the statistical inference about the stress-strength reliability parameter. For further studies on the statistical inference of the stress-strength reliability based on RSS, one can refer to Muttlak et al. (2010), Mahdizadeh (2018), Safariyan et al. (2019a), Safariyan et al. (2019b), Zamanzade et al. (2020), Sadeghpour et al. (2020), Basikhasteh et al. (2020), Esemen et al. (2021), Abdallah et al. (2022), Hassan (2022) and Biradar (2022). In this paper, the point estimations of the stress-strength reliability are given under the classical and Bayesian estimation approaches with RSS.

The remainder of this paper has been organized in the following manner. The maximum likelihood estimation (MLE) and the Bayes estimation of R with SRS and RSS methods are presented in Section 2. In Section 3, a simulation study is carried out to evaluate the estimators of R in detail. Section 4 includes a brief summary.

2 Estimation of stress-strength reliability

Let X be a random variable that follow from the Levy distribution with parameter σ_x and Let Y be a random variable from the Levy distribution with parameter σ_y. Now, we consider a system with random strength Y subjected to a random stress X. Furthermore, assume that the random variables X and Y are independent. From Lemma 1 of Ali and Woo (2005), the stress-strength reliability of this system is obtained by

$$\begin{aligned} R &= P(X < Y) \\ &= \int_0^\infty P(Y > X|X = x) f(x; \sigma_x) dx \\ &= 2\int_0^\infty \Phi\left(\sqrt{\frac{\sigma_y}{x}}\right) f(x; \sigma_x) dx - 1 \\ &= 1 - \frac{2\sqrt{\rho}}{\pi(1+\rho)} F\left(1, 1; \frac{3}{2}; \frac{1}{1+\rho}\right), \end{aligned} \tag{2}$$

where $\rho = \frac{\sigma_y}{\sigma_x}$ and $F(a, b; c; x)$ is the hypergeometric function.

In this section, we provide two common estimators namely the ML and Bayes estimators for the stress-strength reliability R in (2) with SRS and RSS methods.

2.1 ML estimation of R based on SRS

Here, we will find the ML estimator of R based on simple random sampling. Let $X_1, \ldots, X_{n_x}$ be a random sample of size n_x from X with PDF in (1) and $Y_1, \ldots, Y_{n_y}$ be a simple random sample of size n_y from levy distribution with parameter σ_y. Then, Ali and Woo (2005) computed R from (2) as follows:

$$R_{SRS} = 1 - \frac{2}{\pi} \text{Arcsin}\left(\frac{1}{\sqrt{1 + \frac{\sigma_y}{\sigma_x}}}\right). \tag{3}$$

By the invariance property, maximum likelihood estimator of R is

$$\hat{R}_{SRS}^{ML} = 1 - \frac{2}{\pi} \text{Arcsin}\left(\frac{1}{\sqrt{1 + \frac{\hat{\sigma}_{y,SRS}}{\hat{\sigma}_{x,SRS}}}}\right),$$

where $\hat{\sigma}_{x,SRS} = n_x / \sum_{i=1}^{n_x} \frac{1}{X_i}$ and $\hat{\sigma}_{y,SRS} = n_y / \sum_{i=1}^{n_y} \frac{1}{Y_i}$. We have not a closed form for the MSE of $\hat{R}^{ML}_{SRS}$, so numerical computations are needed.

2.2 ML estimation of R based on RSS

Let $X_1, \ldots, X_{m_x}$ be samples from RSS in one cycle from Levy distribution. The probability distribution function of i-th order statistics is given by (David and Nagaraja, 2003)

$$g_i(x_{ij}; \sigma_x) = i \binom{m_x}{i} [F(x_{ij}; \sigma_x)]^{(i-1)} [F(x_{ij}; \sigma_x)]^{(m_x - i)} f(x_{ij}; \sigma_x),$$

where, $i = 1, \ldots, m_x$ and $j = 1, \ldots, r_x$. We recall that m_x and r_x are sample sizes in one cycle and number of cycles, respectively. So, the sample size is $n_x = m_x * r_x$. Also, in the case of random strength Y, we have

$$g_t(y_{ts}; \sigma_y) = t \binom{m_y}{t} [F(y_{ts}; \sigma_y)]^{(t-1)} [F(y_{ts}; \sigma_y)]^{(m_y - i)} f(y_{ts}; \sigma_y),$$

where, $t = 1, \ldots, m_y$ and $s = 1, \ldots, r_y$ are sample size in one cycle and number of cycles, respectively. Consequently, the sample size is $n_y = m_y * r_y$. Therefore, the likelihood function of the samples extracted by RSS method is as follows

$$\begin{aligned} L_{RSS}(\sigma_x, \sigma_y | X, Y) &= \prod_{j=1}^{r_x} \prod_{i=1}^{m_x} g_i(x_{ij}; \sigma_x) \prod_{s=1}^{r_y} \prod_{t=1}^{m_y} g_t(y_{ts}; \sigma_y) \\ &= \left[i \binom{m_x}{i} \right]^{n_x} \left[t \binom{m_y}{t} \right]^{n_y} \left(\frac{\sigma_x}{2\pi} \right)^{\frac{n_x}{2}} \left(\frac{\sigma_y}{2\pi} \right)^{\frac{n_y}{2}} \prod_{j=1}^{r_x} \prod_{i=1}^{m_x} x_{ij}^{-\frac{3}{2}} \\ &\times \exp\left(-\frac{\sigma_x}{2x_{ij}} \right) \left[2 - 2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}} \right) \right]^{i-1} \left[2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}} \right) - 1 \right]^{(m_x - i)} \\ &\times \prod_{s=1}^{r_y} \prod_{t=1}^{m_y} y_{ts}^{-\frac{3}{2}} \exp\left(-\frac{\sigma_y}{2y_{ts}} \right) \left[2 - 2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}} \right) \right]^{t-1} \\ &\times \left[2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}} \right) - 1 \right]^{(m_y - t)}, \end{aligned}$$

by taking Logarithm, we have

$$\begin{aligned} \ell_{RSS}(\sigma_x, \sigma_y | X, Y) \quad = \quad & \log K + \frac{n_x}{2} \log\left(\frac{\sigma_x}{2\pi} \right) + \frac{n_y}{2} \log\left(\frac{\sigma_y}{2\pi} \right) \\ & - \frac{3}{2} \sum_{j=1}^{r_x} \sum_{i=1}^{m_x} \log(x_{ij}) - \frac{3}{2} \sum_{s=1}^{r_y} \sum_{t=1}^{m_y} \log(y_{ts}) \\ & - \frac{\sigma_x}{2} \sum_{j=1}^{r_x} \sum_{i=1}^{m_x} \frac{1}{x_{ij}} - \frac{\sigma_y}{2} \sum_{s=1}^{r_y} \sum_{t=1}^{m_y} \frac{1}{y_{ts}} \\ & + \sum_{j=1}^{r_x} \sum_{i=1}^{m_x} (i-1) \log\left[2 - 2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}} \right) \right] \end{aligned}$$

$$+\sum_{s=1}^{r_y}\sum_{t=1}^{m_y}(t-1)\log\left[2-2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)\right]$$
$$+\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}(m_x-i)\log\left[2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}}\right)-1\right]$$
$$+\sum_{s=1}^{r_y}\sum_{t=1}^{m_y}(m_y-i)\log\left[2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)-1\right], \quad (4)$$

where $K=\left[i\binom{m_x}{i}\right]^{n_x}\left[t\binom{m_y}{t}\right]^{n_y}$. Taking the first order partial derivatives with respect to σ_x and σ_y and setting them to zero, we get the following system of score equations

$$\begin{cases}\frac{\partial \ell_{RSS}(\sigma_x,\sigma_y|X,Y)}{\partial\sigma_x}=0,\\ \frac{\partial \ell_{RSS}(\sigma_x,\sigma_y|X,Y)}{\partial\sigma_y}=0.\end{cases}$$

So,

$$\begin{cases}\frac{n_x}{2\sigma_x}-\frac{1}{2}\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{1}{x_{ij}}-\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}(i-1)\frac{\frac{1}{\sqrt{\sigma_x x_{ij}}}\varphi\left(\sqrt{\frac{\sigma_x}{x_{ij}}}\right)}{\left[2-2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}}\right)\right]}\\ \quad+\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}(m_x-i)\frac{\frac{1}{\sqrt{\sigma_x x_{ij}}}\varphi\left(\sqrt{\frac{\sigma_x}{x_{ij}}}\right)}{\left[2-2\Phi\left(\sqrt{\frac{\sigma_x}{x_{ij}}}\right)\right]}=0,\\ \frac{n_y}{2\sigma_y}-\frac{1}{2}\sum_{s=1}^{r_y}\sum_{t=1}^{m_y}\frac{1}{y_{ts}}-\sum_{s=1}^{r_y}\sum_{t=1}^{m_y}(t-1)\frac{\frac{1}{\sqrt{\sigma_y y_{ts}}}\varphi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)}{\left[2-2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)\right]}\\ \quad+\sum_{s=1}^{r_y}\sum_{t=1}^{m_y}(m_y-t)\frac{\frac{1}{\sqrt{\sigma_y y_{ts}}}\varphi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)}{\left[2-2\Phi\left(\sqrt{\frac{\sigma_y}{y_{ts}}}\right)\right]}=0.\end{cases} \quad (5)$$

The MLEs of σ_x and σ_y based on RSS denoted by $\hat{\sigma}_{x,RSS}$ and $\hat{\sigma}_{y,RSS}$. Also, $\varphi(.)$ is the PDF of normal distribution. The solutions of the (5) system of score equations maximized the likelihood function (4). By solving the system of non-linear equations in (5), $\hat{\sigma}_{x,SRS}$ and $\hat{\sigma}_{y,SRS}$ will be obtained. By the invariance property of ML estimators, $\hat{R}^{ML}_{RSS}$ can be obtained by

$$\hat{R}^{ML}_{RSS}=1-\frac{2}{\pi}\text{Arcsin}\left(\frac{1}{\sqrt{1+\frac{\hat{\sigma}_{y,RSS}}{\hat{\sigma}_{x,RSS}}}}\right).$$

2.3 Bayesian estimation of R based on SRS

For Bayes estimation of R, suppose the scale parameters, i.e., σ_x and σ_y satisfy the following gamma priors

$$\sigma_x\sim GAM\left(\frac{\alpha_x}{2},\lambda_x\right), \qquad \sigma_y\sim GAM\left(\frac{\alpha_y}{2},\lambda_y\right).$$

The posterior distributions of σ_x and σ_y are

$$\pi(\sigma_x|\boldsymbol{x})\sim GAM\left(\frac{n_x+\alpha_x}{2},\ \lambda_x+\frac{1}{2}\sum_{i=1}^{n_x}\frac{1}{X_i}\right),$$

$$\pi\left(\sigma_y|\boldsymbol{y}\right) \sim GAM\left(\frac{n_y+\alpha_y}{2},\ \lambda_y+\frac{1}{2}\sum_{i=1}^{n_y}\frac{1}{Y_i}\right).$$

Under the mean squared error loss function, the Bayes estimator of R is

$$\hat{R}_{SRS}^{Bayes} = \int_0^1 rf\left(r|x,y\right)dr,$$

where, $f\left(r|x,y\right)$ is the posterior distribution of R. Najarzadegan et al. (2016) show that analytical solutions for the above integral are not available. Lindley's approximation is an approximate approach to calculate the ratio of two integrals. In the following, the Lindley's approximation method is used for Bayes estimators.

We consider Lindley's approximation (Lindley, 1980) form expanding about the posterior mode. For two parameters case $\theta=(\theta_1,\theta_2)$, Lindley's approximation leads to

$$\hat{U}_{Lindley} = \left(U(\theta)+\frac{1}{2}[B+Q_{30}B_{12}+Q_{21}C_{12}+Q_{12}C_{21}+Q_{03}B_{21}]\right)|_{(\theta_1,\theta_2)=(\tilde{\theta}_1,\tilde{\theta}_2)}, \quad (6)$$

where, $B=\sum_{i=1}^{2}\sum_{j=1}^{2}U_{ij}\tau_{ij}$, $Q_{\eta\xi}=\frac{\partial^{\eta+\xi}}{\partial^{\eta}\theta_1\partial^{\xi}\theta_2}$, $\eta,\xi=0,1,2,3$, $\eta+\xi=3$, for $i,j=1,2$, $U_i=\frac{\partial U}{\partial\theta_i}$, and for $i\neq j$, $U_{ij}=\frac{\partial^2 U}{\partial\theta_i\partial\theta_j}$, $C_{ij}=3U_i\tau_{ii}\tau_{ij}+U_j(\tau_{ii}\tau_{ij}+2\tau_{ij}^2)$, $B_{ij}=(U_i\tau_{ii}+U_j\tau_{ij})\tau_{ii}$ $C_{ij}=3U_i\tau_{ii}\tau_{ij}+U_j(\tau_{ii}\tau_{ij}+2\tau_{ij}^2)$, where τ_{ij} is the (i,j)th element in the inverse of matrix $Q^*=(-Q_{ij}^*), i,j=1,2$ such that $Q_{ij}^*=\frac{\partial^2 Q}{\partial\theta_i\partial\theta_j}$. In fact, Q is the logarithm of the posterior density function and $(\tilde{\theta}_1,\tilde{\theta}_2)$ is the mode of the posterior density function. In our case, $(\theta_1,\theta_2)=(\sigma_1,\sigma_2)$. The elements of Lindley's approximation in (6) can be given bellow,

$$\begin{aligned}
U(\theta) &= \frac{2}{\pi}Arcsin\left(\frac{1}{\sqrt{1+\frac{\sigma_1}{\sigma_2}}}\right),\\
U_1 &= \frac{\sqrt{\frac{\sigma_1}{\sigma_2}}}{\pi(\sigma_1+\sigma_2)},\\
U_2 &= \frac{1}{\pi\sigma_1\sqrt{\frac{\sigma_2}{\sigma_1+\sigma_2}}(\frac{\sigma_1+\sigma_2}{\sigma_1})^{\frac{3}{2}}},\\
Q_{30} &= \frac{\partial^3 Q}{\partial^3\sigma_1}=\frac{2(\frac{n_1}{2}+\frac{\alpha_1}{2}-1)}{S_1^3}-2\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{e^{-\frac{\sigma_1}{2x_{ij}}}(\sigma_1^2+2\sigma_1x_{ij}+3x_{ij}^2)}{8\sqrt{2\pi}(\frac{\sigma_1}{x_{ij}})^{\frac{5}{2}}x_{ij}^5}\\
&\quad +2\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}(m-i)\frac{e^{-\frac{\sigma_1}{2x_{ij}}}(\sigma_1^2+2\sigma_1x_{ij}+3x_{ij}^2)}{8\sqrt{2\pi}(\frac{\sigma_1}{x_{ij}})^{\frac{5}{2}}x_{ij}^5},
\end{aligned}$$

$Q_{12}=Q_{21}=0$. By substituting the above values in (6), the Lindley's approximation of Bayes estimator have been obtained.

2.4 Bayesian estimation of R based on RSS

If the sampling scheme is RSS, we use the log-likelihood function in (4). The Bayes estimate of R based on RSS is denoted by $\hat{R}_{RSS}^{Bayes}$. In the following section, $\hat{R}_{SRS}^{Bayes}$ and $\hat{R}_{RSS}^{Bayes}$ are calculated and compared for the different parameters.

3 Simulation study

In this section, we accomplish a simulation study to compare the performances of the estimators of R. Due to the estimators are not comparable analytically, we have compared them via a Monte-Carlo simulation. For this mean, we have used some criteria such as biased, mean square error (MSE), and relative efficiency (RE) that are given by $Bias\left(\hat{R}\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left(\hat{R}_i - R\right)$, $MSE\left(\hat{R}\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left(\hat{R}_i - R\right)^2$, and $RE = \frac{MSE\left(\hat{R}_{SRS}\right)}{MSE\left(\hat{R}_{RSS}\right)}$.

In Tables 1-4, for some values of n, r, m, σ_x, and σ_y, the estimated values of the parameter R, MSE and RE are reported. Tables 1-4 show that by increasing the values of n, the values of MSE decrease in both SRS and RSS methods. It is better to consider the number of cycles as little as possible to reduce the MSE. We also see that in all cases the relative efficiency is more than one, which indicates the superiority of the RSS method over the SRS method for estimating R. It is observed that if the number of cycles is small, the relative efficiency is higher. It should be noted that in the case of $\sigma_x = \sigma_y$, the value of stress-strength reliability is equal to $\frac{1}{2}$. As it can be seen from Tables 1-4, ML estimates of R are biased. Also, it can be concluded from present results that estimates of R based on RSS are closer to exact amount of R in a majority of cases than the corresponding estimates using SRS for ML estimation.

Table 1: The values of estimations of R, ER, and MSE for $\sigma_x = \sigma_y = 1$, whereas the exact amount of $R = 0.5$.

n	r, m	$\hat{R}_{SRS}^{ML}$	$\hat{R}_{RSS}^{ML}$	$MSE(\hat{R}_{SRS}^{ML})$	$MSE(\hat{R}_{RSS}^{ML})$	RE
20	5, 4	0.4997	0.4993	0.0051	.0024	2.0817
	4, 5	0.4993	0.5005	0.0049	0.0020	2.4386
	2, 10	0.4990	0.4998	0.0049	0.0012	4.0062
30	6, 5	0.5005	0.4996	0.0033	0.0013	2.3905
	5, 6	0.4997	0.4999	0.0035	0.0012	2.9060
	3, 10	0.5001	0.5001	0.0033	0.0008	4.1626
40	8, 5	0.5001	0.5002	0.0025	0.0010	2.4452
	5, 8	0.5001	0.5001	0.0024	0.0007	3.5009
	4, 10	0.5005	0.5004	0.0025	0.0006	4.0922

We study the effect of prior parameters on Bayes estimators with RSS and SRS methods. First for a given vector of parameters $(\alpha_x, \lambda_x, \alpha_y, \lambda_y)$ in Table 5, which includes least informative, informative and most informative, for $N = 10^3$ replication, we generate σ_x and σ_y from the prior distributions. For the given vector of parameters $(\alpha_x, \lambda_x, \alpha_y, \lambda_y)$, the Bayes estimations based on RSS and SRS methods are computed and reported in Tables 6-13. Tables 6-13 show that by increasing informative (changes

Table 2: The values of estimations of R, ER, and MSE for $\sigma_x = 1$ and $\sigma_y = 5$, whereas the exact amount of $R = 0.2685$.

n	r, m	$\hat{R}^{ML}_{SRS}$	$\hat{R}^{ML}_{RSS}$	$MSE(\hat{R}^{ML}_{SRS})$	$MSE(\hat{R}^{ML}_{RSS})$	RE
20	5, 4	0.2714	0.2695	0.0029	0.0014	2.1042
	4, 5	0.2720	0.2697	0.0030	0.0012	2.5466
	2, 10	0.2717	0.2682	0.0029	0.0006	4.3806
30	6, 5	0.2702	0.2687	0.0019	0.0008	2.4031
	5, 6	0.2699	0.2685	0.0019	0.0007	2.8467
	3, 10	0.2702	0.2681	0.0019	0.0004	4.2324
40	8, 5	0.2696	0.2685	0.00014	0.0006	2.4742
	5, 8	0.2694	0.2683	0.0014	0.0004	3.5563
	4, 10	0.2699	0.2682	0.0015	0.0003	4.4420

Table 3: The values of estimations of R, ER, and MSE for $\sigma_x = \sigma_y = 10$, whereas the exact amount of $R = 0.5$.

n	r, m	$\hat{R}^{ML}_{SRS}$	$\hat{R}^{ML}_{RSS}$	$MSE(\hat{R}^{ML}_{SRS})$	$MSE(\hat{R}^{ML}_{RSS})$	RE
20	5, 4	0.5002	0.4997	0.0052	0.0024	2.1480
	4, 5	0.4994	0.4998	0.0050	0.0020	2.4657
	2, 10	0.5005	0.5002	0.0049	0.0012	4.1266
30	6, 5	0.5002	0.4998	0.0033	0.0014	2.4134
	5, 6	0.4997	0.5002	0.0033	0.0012	2.6955
	3, 10	0.4998	0.5000	0.0034	0.0008	4.1592
40	8, 5	0.4996	0.4991	0.0025	0.0010	2.4410
	5, 8	0.4999	0.4997	0.0025	0.0007	3.5064
	4, 10	0.5003	0.4996	0.0025	0.0006	4.0387

Table 4: The values of estimations of R, ER, and MSE for $\sigma_x = 10$ and $\sigma_y = 5$, whereas the exact amount of $R = 0.6071$.

n	r, m	$\hat{R}^{ML}_{SRS}$	$\hat{R}^{ML}_{RSS}$	$MSE(\hat{R}^{ML}_{SRS})$	$MSE(\hat{R}^{ML}_{RSS})$	RE
20	5, 4	0.6055	0.6057	0.0046	0.0022	2.0601
	4, 5	0.6053	0.6067	0.0044	0.0018	2.4337
	2, 10	0.6062	0.6080	0.0045	0.0011	4.1659
30	6, 5	0.6069	0.6077	0.0030	0.0012	2.4131
	5, 6	0.6057	0.6073	0.0030	0.0011	2.7842
	3, 10	0.6058	0.6082	0.0030	0.0007	4.1164
40	8, 5	0.6067	0.6074	0.0023	0.0009	2.4609
	5, 8	0.6064	0.6080	0.0023	0.0007	3.4903
	4, 10	0.6069	0.6077	0.0023	0.0006	4.0433

α and λ), the RE increased. Also, it is observed that the ER is sensitive with respect to prior parameters, and also is decreased as the sample size increases. In addition, Bayesian estimators based on RSS have much lower MSE than SRS.

Table 5: Information in the case of constant mean.

Information	α	λ	$E(\sigma)$	$Var(\sigma)$
Least informative	2	0.5	0.5	0.25
Informative	10	0.1	0.5	0.05
Most informative	20	0.05	0.5	0.005

Table 6: The values of estimations of R and ER for $\sigma_x = \sigma_y = 1$ and $n = 20$.

Information	r, m	$\hat{R}_{SRS}^{Bayes}$	$\hat{R}_{RSS}^{Bayes}$	RE
Least informative	5, 4	0.4995	0.5004	3.2863
	4, 5	0.4987	0.4995	4.4428
	2, 10	0.5007	0.4992	7.3259
Informative	5, 4	0.4982	0.5000	5.2147
	4, 5	0.5010	0.4996	5.8489
	2, 10	0.4996	0.4994	11.2314
Most informativ	5, 4	0.5005	0.4996	6.3837
	4, 5	0.4999	0.5001	7.4843
	2, 10	0.4993	0.5000	14.1972

Table 7: The values of estimations of R and ER for $\sigma_x = 1$, $\sigma_y = 5$, and $n = 20$.

Information	r, m	$\hat{R}_{SRS}^{Bayes}$	$\hat{R}_{RSS}^{Bayes}$	RE
Least informative	5, 4	0.2683	0.2680	3.6579
	4, 5	0.2703	0.2671	4.5764
	2, 10	0.2695	0.2680	8.0059
Informative	5, 4	0.2684	0.2674	4.8588
	4, 5	0.2686	0.2681	6.0181
	2, 10	0.2677	0.2676	11.6597
Most informativ	5, 4	0.2686	0.2677	5.8976
	4, 5	0.2682	0.2676	7.7409
	2, 10	0.2676	0.2678	13.9536

Table 8: The values of estimations of R and ER for $\sigma_x = \sigma_y = 10$ and $n = 20$.

Information	r, m	$\hat{R}_{SRS}^{Bayes}$	$\hat{R}_{RSS}^{Bayes}$	RE
Least informative	5, 4	0.4975	0.4995	3.1768
	4, 5	0.5012	0.5002	4.1980
	2, 10	0.4996	0.4995	7.7028
Informative	5, 4	0.4995	0.5002	5.1936
	4, 5	0.5001	0.4997	5.0812
	2, 10	0.5006	0.4996	11.8719
Most informativ	5, 4	0.5003	0.5000	5.8489
	4, 5	0.4992	0.4998	7.7083
	2, 10	0.4997	0.5001	15.8817

4 Conclusion and summary

In this study, the stress-strength reliability with independent stress and strength random variables from the levy distribution function is investigated. This is estimated using the ML and the Bayesian approaches under both SRS and RSS methods. The Bayesian estimates o are achieved by using Lindley's approximation method. The simulation results indicate that the estimate of stress-strength reliability using RSS is closer to actual value than the corresponding estimate based on SRS for both estimation methods in the majority of the cases. Both estimates are biased. Furthermore, as specified by the RE values, it can be seen that RSS gives more efficient results than

Table 9: The values of estimations of R and ER for $\sigma_x = 10$, and $\sigma_y = 5$, and $n = 20$.

Information	r, m	$\hat{R}^{Bayes}_{SRS}$	$\hat{R}^{Bayes}_{RSS}$	RE
Least informative	5, 4	0.6069	0.6077	3.4211
	4, 5	0.6078	0.6081	4.1679
	2, 10	0.6073	0.6084	7.1177
Informative	5, 4	0.6069	0.6079	4.6815
	4, 5	0.6079	0.6081	6.5089
	2, 10	0.6094	0.6081	11.7775
Most informativ	5, 4	0.6075	0.6083	6.1685
	4, 5	0.6079	0.6080	7.2933
	2, 10	0.6085	0.6084	14.4869

Table 10: The values of estimations of R and ER for $\sigma_x = \sigma_y = 1$ and $n = 40$.

Information	r, m	$\hat{R}^{Bayes}_{SRS}$	$\hat{R}^{Bayes}_{RSS}$	RE
Least informative	8, 5	0.4992	0.4994	3.8820
	5, 8	0.4993	0.4999	6.4046
	4, 10	0.5004	0.5002	6.8994
Informative	8, 5	0.4996	0.4997	5.4652
	5, 8	0.5004	0.5000	9.6104
	4, 10	0.5008	0.4999	11.8274
Most informativ	8, 5	0.4992	0.4998	7.3319
	5, 8	0.4997	0.5000	9.8745
	4, 10	0.5002	0.4998	15.1474

Table 11: The values of estimations of R and ER for $\sigma_x = 1$, $\sigma_y = 5$, and $n = 40$.

Information	r, m	$\hat{R}^{Bayes}_{SRS}$	$\hat{R}^{Bayes}_{RSS}$	RE
Least informative	8, 5	0.2683	0.2676	4.1670
	5, 8	0.2683	0.2680	6.0566
	4, 10	0.2693	0.2675	8.6132
Informative	8, 5	0.2679	0.2677	5.9798
	5, 8	0.2674	0.2677	9.1003
	4, 10	0.2676	0.2676	10.5399
Most informativ	8, 5	0.2680	0.2677	8.4534
	5, 8	0.2678	0.2679	10.8632
	4, 10	0.2681	0.2677	15.3346

Table 12: The values of estimations of R and ER for $\sigma_x = \sigma_y = 10$ and $n = 40$.

Information	r, m	$\hat{R}^{Bayes}_{SRS}$	$\hat{R}^{Bayes}_{RSS}$	RE
Least informative	8, 5	0.4994	0.4997	4.7329
	5, 8	0.5002	0.4998	6.1252
	4, 10	0.4995	0.5005	6.8205
Informative	8, 5	0.4998	0.5000	5.7612
	5, 8	0.4997	0.5002	8.5206
	4, 10	0.4999	0.5002	11.4454
Most informativ	8, 5	0.4997	0.5001	7.5568
	5, 8	0.4998	0.5001	11.5534
	4, 10	0.4994	0.5000	14.6397

Table 13: The values of estimations of R and ER for $\sigma_x = 10$, $\sigma_y = 5$, and $n = 40$.

Information	r, m	$\hat{R}_{SRS}^{Bayes}$	$\hat{R}_{RSS}^{Bayes}$	RE
Least informative	8, 5	0.6077	0.6084	4.6946
	5, 8	0.6076	0.6082	6.0137
	4, 10	0.6084	0.6082	7.4957
Informative	8, 5	0.6087	0.6083	5.5094
	5, 8	0.6082	0.6081	9.2611
	4, 10	0.6095	0.6079	11.1944
Most informativ	8, 5	0.6077	0.6083	7.0069
	5, 8	0.6083	0.6082	11.5227
	4, 10	0.6088	0.6081	13.8289

SRS for both ML and Bayesian methods. In the continuation of the work done, the obtained estimators can be checked in the case that there is an error in the ranking of sample units or in the case that the sampling plane of the ranked set is unbalanced. On the other hand, considering the various generalizations that exist for the ranked set sampling plan, it is possible to examine the estimators in the new plans and compare the efficiency of these methods with other existing sampling methods.

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