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Reliability analysis of weighted-(r, s)-out-of-n system having two dependent subcomponents each with its own positive integer-valued weight

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Abstract: In this paper, a general form of weighted-(r, s)-out-of-n system having two dependent subcomponents with different weights is introduced. This system consists of n independent components, each having two dependent subcomponents, each with its own positive integer-valued weight. If the total weights of the working subcomponents exceed a pre-specified threshold (r, s), the system is assumed to function. The reliability and component importance of this system are studied. The survival function and mean time to failure are presented.

Keywords: Component importance; Dependence; Path set; Reliability; Weighted-(r, s)-out-of-n system.
Mathematics Subject Classification (2010): 62N05, 90B25.

1 Introduction

In the literature, the coherent systems consisting of n single (independent/dependent) components have been extensively studied and their reliability analysis and all properties are discussed through various approaches. In all these considered systems, the components are single elements. But, in some practical applications, engineers encounter systems consisting of components with two or more subcomponents. The (r, s)-out-of-n systems were introduced by Bairamov (2013) and the reliability and mean residual life functions of some complex systems consisting of n elements are studied which each element having two s-dependent components. This system is assumed to function if and only if at least r of n first subcomponents and at least s of n second subcomponents

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work. Eryilmaz (2017) introduced systems having two dependent subcomponents and investigated its reliability and properties for a consecutive-(k, k)-out-of-n:F system. Assume that this system works if and only if both the system of first subcomponents and the system of second subcomponents work under certain structural rules. Based on the Birnbaum measure, Estabraqi and Meshkat (2021) investigated the importance of components in a consecutive-(k, k)-out-of-n:F system having two dependent subcomponents.

In most real life systems, the total contribution of the components plays an important role in the system's performance, which must be above a specified performance level, and this contribution might be different for each component. Wu and Chen (1994) introduced the weighted systems with unequal weights which to deal with the studied situation in the literature. A system including n components with their different positive integer weights is known as weighted-k-out-of-n:G system when it works if and only if the total weight of working components is above a threshold k. A review of weighted-k-out-of-n systems is presented by Samaniego and Shaked (2008). Li and Zuo (2008) provided two models of multi-state weighted-k-out-of-n system models and presented the recursive algorithms for their reliability evaluation. Navarro et al. (2011) extended the signature-based representations of the reliability functions of coherent systems for the systems with heterogeneous components. Eryilmaz and Bozbulut (2014) studied a multi-state weighted-k-out-of-n:G system model in a dynamic setup and yielded an algorithmic approach to its dynamic reliability analysis. Eryilmaz and Sarikaya (2014) studied the special case of weighted-k-out-of-n: G system containing two types of components, each group having different weights and reliabilities and also got the non-recursive equations for the system reliability, survival function and Mean Time To Failure (MTTF). Eryilmaz (2019) introduced the $(k_1, k_2, \cdots, k_m)$-out-of-n system including n_i components of type i for $i = 1, \cdots, m$ and $n = \sum_{i=1}^{m} n_i$. Mahmoudi and Meshkat (2020) investigated the reliability analysis and component importance of weighted-k-out-of-n: G system consisting of two different types with non-identical components. Mahmoudi et al. (2021) studied the copula-based reliability of weighted-k-out-of-n systems consisting of m types of dependent components which chosen randomly.Hamdan et al. (2023) provided the reliability function of a weighted k-out-of-n system with multiple types of components considering two real-life scenarios for the operation of the system.

To assess the change in system reliability relating to the change in the reliability of that component, ranking the components according to their importance measure has an important role the evaluation of reliabilities of systems at various stages. In reliability literature, various measures of importance have been proposed by Barlow and Proschan (1975), Birnbaum (1969), Boland and El-Neweihi (1995), Kuo and Zhu (2012), Wu and Coolen (2013), Zhu and Kuo (2014), Rahmani et al. (2016) and Meshkat and Mahmoudi (2017).

In this study, we consider a general setup of weighted-(r, s)-out-of-n system consisting n independent components, each having two dependent subcomponents each with its own positive integer-valued weight. In the proposed setup, a specific weight is assigned to each subcomponent of a component. If the total weights of the working subcomponents exceed a pre-specified threshold (r, s), the system is assumed to function. In our defined system, the sum of the functioning first (second) subcomponents weights must be at least r (s). Our introduced system can be considered an extension of the systems defined by Eryilmaz (2017). The rest of the paper is organized as follows. In Section 2, for the general setup of weighted-(r,s)-out-of-n system, the description of the modelling system is provided. The reliability evaluation of this system is studied in Section 3 and also, the Birnbaum reliability importance is investigated. In Section 4, the survival function and MTTF are obtained. Also, some illustrative examples are presented to evaluate the results in section 3 and 4. Finally, concluding remarks are given in Section 5.

2 Model description

In this section, first some notations and the description of modelling system are provided for this general setup of weighted-(r,s)-out-of-n system.

Notations

n	Number of components in a system
$A_i(B_i)$	First (Second) subcomponent of i-th component of the system
$X_i(Y_i)$	State of i-th first (second) subcomponent of i-th component which is 1 if subcomponent functions and 0 if it fails
r (s)	Minimum total weight (capacity) of all working first (second) subcomponent to operate system
ω_i	Weight of i-th first subcomponent
ω_i^*	Weight of i-th second subcomponent
$p_{11}(p_{00})$	probability that i-th first subcomponent A_i and i-th second subcomponent B_i work (fail)
$p_{10}(p_{01})$	probability that i-th first subcomponent A_i works (fails) and i-th second subcomponent B_i fails (works)

Consider a system including n independent components, each having two dependent subcomponents $(A_i, B_i), i = 1, \cdots, n$. The bivariate vectors $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ denote the state of components $(A_1, B_1), (A_2, B_2), \cdots, (A_n, B_n)$. The weighted-$(r,s)$-out-of-$n$ system is assumed to function if and only if the total weights of the working first subcomponents is above a pre-specified threshold r and the total weights of the working second subcomponents is above a pre-specified threshold s. Now, since in our defined setup, the system operates if the total weights of the working subcomponents exceed the threshold (r,s), the structure function of such a system is given by

$$\phi(X_1, Y_1, \cdots, X_n, Y_n) = \begin{cases} 1, & \sum_{i=1}^{n} \omega_i X_i \geq r \text{ and } \sum_{i=1}^{n} \omega_i^* Y_i \geq s, \\ 0, & o.w. \end{cases} \tag{1}$$

3 Main results

In this section, by considering independent and identical components for a weighted-(r,s)-out-of-n system and using the definition of path sets, the reliability function and importance component are obtained.

3.1 Reliability evaluation

Consider a weighted-(r,s)-out-of-n system including n independent components each having two dependent subcomponents $(A_1,B_1),(A_2,B_2),\cdots,(A_n,B_n)$. Having the bivariate state vectors $(X_1,Y_1),(X_2,Y_2),\cdots,(X_n,Y_n)$, the probabilities of subcomponents state are as following

$$\begin{aligned} p_{00} &= \mathrm{P}(X_i=0,Y_i=0),\\ p_{10} &= \mathrm{P}(X_i=1,Y_i=0),\\ p_{01} &= \mathrm{P}(X_i=0,Y_i=1),\\ p_{11} &= \mathrm{P}(X_i=1,Y_i=1),\end{aligned}$$

where $p_{00}+p_{10}+p_{01}+p_{11}=1$. Note that in the reliability analysis, the dependence of the subcomponents is taken into account through the joint probabilities p_{00},p_{10},p_{01} and p_{11}.

If the number of working subcomponents $A_1,\cdots,A_n$ is $N(X)=\sum_{i=1}^n X_i$ and the number of working subcomponents $B_1,\cdots,B_n$ is $N(Y)=\sum_{i=1}^n Y_i$, then the vector $\big(N(X),N(Y)\big)$ has bivariate binomial distribution. Before continuing the discussion, a brief description of a bivariate binomial model is represented; see Johnson et al. (1997). Let a population be classified as either E or E^c and simultaneously as either F or F^c with the probabilities

	E	E^c
F	p_{11}	p_{01}
F^c	p_{10}	p_{00}

Under random sampling with replacement n times, the random variables $N(E)$ and $N(F)$ denoting the numbers of occurrences of E and F, respectively, have jointly the following probability mass function

$$\begin{aligned}\mathrm{P}\big(N(E)=i,N(F)=j\big) &= \sum_{l=\max(0,i+j-n)}^{\min(i,j)} \frac{n!}{l!(i-l)!(j-l)!(n-i-j+l)!}\\ &\qquad \times p_{11}^{l}p_{10}^{i-l}p_{01}^{j-l}p_{00}^{n-i-j+l}.\end{aligned}$$

This bivariate discrete distribution is called a bivariate binomial distribution.

So the joint probability of $(N(X),N(Y))$ is given by

$$\begin{aligned}\mathrm{P}\big(N(X)=k_1,N(Y)=k_2\big) &= \sum_{a=\max(0,k_1+k_2-n)}^{\min(k_1,k_2)} \frac{n!}{a!(k_1-a)!(k_2-a)!(n-k_1-k_2+a)!}\\ &\qquad \times p_{11}^{a}p_{10}^{k_1-a}p_{01}^{k_2-a}p_{00}^{n-k_1-k_2+a}. \qquad (2)\end{aligned}$$

Suppose that $r_n(k_1,k_2,a)$ is the number of path sets of the system which include k_1 working subcomponents from $A_1,\cdots,A_n$, k_2 working subcomponents from $B_1,\cdots,B_n$, and a total of components with both working subcomponents. The reliability of the system with the structure function (1) is obtained as

$$R=\mathrm{P}\big(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1\big)=\mathrm{P}\left(\sum_{i=1}^n \omega_i X_i \geq r\ ,\ \sum_{i=1}^n \omega_i^* Y_i \geq s\right). \qquad (3)$$

Now, let $N(XY) = \sum_{i=1}^{n} X_i Y_i$ denotes the number of components that both of its subcomponents work and according by the properties of bivariate binomial distribution. By Remark 1 Eryilmaz (2017), the number $r_n(k_1, k_2, a)$ coincides with the number of bivariate sequences $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ such that $\sum_{i=1}^{n} X_i = k_1$, $\sum_{i=1}^{n} Y_i = k_2$, $\sum_{i=1}^{n} X_i Y_i = a$, $\phi_1(X_1, \cdots, X_n) = 1$ and $\phi_2(Y_1, \cdots, Y_n) = 1$. By the structure function (1) and conditioning on the $(N(X), N(Y), N(XY))$, the reliability of the system can be rewritten as

$$\begin{aligned} R &= \mathrm{P}(\phi(X_1, Y_1, \cdots, X_n, Y_n) = 1) \\ &= \sum_{k_1=1}^{n} \sum_{k_2=1}^{n} \sum_{a=\max(0,k_1+k_2-n)}^{\min(k_1,k_2)} \mathrm{P}\big(\phi(X_1, Y_1, \cdots, X_n, Y_n) = 1 \mid N(X) = k_1, N(Y) = k_2, \\ &\qquad N(XY) = a\big)\mathrm{P}(N(X) = k_1, N(Y) = k_2, N(XY) = a). \quad (4) \end{aligned}$$

Regarding the conditional working probability only depends on the structure of system, this probability is obtained by dividing the number of path sets $r_n(k_1, k_2, a)$ by the combination of all k_1 working subcomponents from $A_1, \cdots, A_n$, k_2 working subcomponents from $B_1, \cdots, B_n$, and a total of components with both working subcomponents, so it is given as

$$\begin{aligned} \mathrm{P}(\phi(X_1, Y_1, \cdots, X_n, Y_n) = 1 \ &| \ N(X) = k_1, N(Y) = k_2, N(XY) = a) \\ &= \frac{r_n(k_1, k_2, a)}{\binom{n}{a, k_1-a, k_2-a, n-k_1-k_2+a}}, \end{aligned} \quad (5)$$

where $r_n(k_1, k_2, a)$ is assessed under the conditions $\sum_{i=1}^{n} \omega_i X_i \geq r$ and $\sum_{i=1}^{n} \omega_i^* Y_i \geq s$. Thus, by substituted (2) and (5) in (4), the reliability of weighted-(r, s)-out-of-n system is calculated as following

$$R = \sum\sum_{(k_1,k_2)\in J} \sum_{a=\max(0,k_1+k_2-n)}^{\min(k_1,k_2)} r_n(k_1, k_2, a) p_{11}^{a} p_{10}^{k_1-a} p_{01}^{k_2-a} p_{00}^{n-k_1-k_2+a}, \quad (6)$$

where $J = \left\{ (k_1, k_2) : k_1, k_2 = 1, \cdots, n, \ \sum_{T_1} \omega_i X_i \geq r \ , \ \sum_{T_2} \omega_i^* Y_i \geq s \right\}$ and T_1 (T_2) extends summation over all k_1 (k_2) combination of $\{1, \cdots, n\}$. Note that for $\omega_i = \omega$ and $\omega_i^* = \omega^*$, the conditions $\sum_{i=1}^{n} \omega_i X_i \geq r$ and $\sum_{i=1}^{n} \omega_i^* Y_i \geq s$ replace by $k_1\omega \geq r$ and $k_2\omega^* \geq s$.

Example 3.1. *Consider a weighted-$(7, 8)$-out-of-5 system with different weights ω_i and ω_i^* and different cases of probabilities of subcomponents state p_{00}, p_{10}, p_{01} and p_{11} as*

p_{00}	p_{10}	p_{01}
0.05	0.1	0.2
0.2	0.15	0.15
0.1	0.63	0.07
0.01	0.09	0.08

C_i	1	2	3	4	5
ω_i	2	1	3	1	2
ω_i^*	3	2	1	3	1

Here under the conditions $\sum_{i=1}^{5} \omega_i X_i \geq 7$ and $\sum_{i=1}^{5} \omega_i^* Y_i \geq 8$, the number of path sets for different components with respect to all possible values of k_1, k_2 and a is as follows.

Table 1: The number of path sets.

k_1	k_2	a	$r_n(k_1, k_2, a)$
3	3	1	1
3	4	2	2
3	4	3	1
3	5	3	1
4	3	2	3
4	3	3	1
4	4	3	10
4	4	4	2
4	5	4	4
5	3	3	1
5	4	4	3
5	5	5	1

In Table 2, the reliability of the weighted-$(7, 8)$-out-of-5 is calculated with respect to the different values of probabilities of subcomponents state. As observed, the system reliability is sensitive to the weights of subcomponents which determine the value of $r_n(k_1, k_2, a)$ under the conditions $\sum_{i=1}^{5} \omega_i X_i \geq 7$ and $\sum_{i=1}^{5} \omega_i^* Y_i \geq 8$. Indeed, when the larger values of working probabilities are assigned to the subcomponents with large weights, the system reliability increases.

Table 2: Reliability of weighted-$(7, 8)$-out-of-5 system.

p_{00}	p_{10}	p_{01}	R
0.05	0.10	0.20	0.4092
0.05	0.20	0.10	0.3922
0.20	0.15	0.15	0.1655
0.10	0.60	0.07	0.0221
0.10	0.07	0.60	0.0224
0.01	0.09	0.08	0.6976
0.01	0.08	0.09	0.7018

3.2 Importance component

Regard to the important role of total contribution of the components in system reliability, the Birnbaum reliability importance is computing in a weighted-(r, s)-out-of-n system with two dependent subcomponents. Birnbaum (1969) introduced the importance of the i-th component in a coherent system. Suppose that the event S shows that the system works. The Birnbaum reliability importance of the i-th component is defined by

$$I_i = \mathrm{P}(S|X_i = 1) - \mathrm{P}(S|X_i = 0).$$

Consider a weighted-(r, s)-out-of-n system with independent components, each having two dependent subcomponents. Suppose that the state of i-th component is $C_i = 1 - (1 - X_i)(1 - Y_i)$ which is 1 if this component works and is 0 if this component fails. The Birnbaum reliability importance of i-th components can be obtained as

$$I_i = \mathrm{P}(S|C_i = 1) - \mathrm{P}(S|C_i = 0)$$

$$\begin{aligned}
&= \Big[\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(X_i,Y_i)=(1,1)) \\
&\quad +\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(X_i,Y_i)=(1,0)) \\
&\quad +\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(X_i,Y_i)=(0,1))\Big] \\
&\quad -\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=0|(X_i,Y_i)=(0,0)). \qquad (7)
\end{aligned}$$

According to the definition of conditional probability and using equation (6), the probabilities are given as following

$$\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(1,1)) = \frac{\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1,(1,1))}{\mathrm{P}(X_i=1,Y_i=1)}$$

$$= \sum_{\substack{\sum_{T_1}\omega_i X_i\geq r\ ,\ \sum_{T_2}\omega_i^* Y_i\geq s \\ 2\leq k_1,k_2\leq n}}\sum \sum_{a=a_L}^{a_U} r_n^{11}(k_1,k_2,a)p_{11}^{a-1}p_{10}^{k_1-a}p_{01}^{k_2-a}p_{00}^{n-k_1-k_2+a},$$

$$\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(1,0))$$

$$= \sum_{\substack{\sum_{T_1}\omega_i X_i\geq r\ ,\ \sum_{T_2}\omega_i^* Y_i\geq s \\ 2\leq k_1\leq n,1\leq k_2\leq n-1}}\sum \sum_{a=a_L}^{a_U} r_n^{10}(k_1,k_2,a)p_{11}^{a}p_{10}^{k_1-a-1}p_{01}^{k_2-a}p_{00}^{n-k_1-k_2+a},$$

$$\mathrm{P}(\phi(X_1,Y_1,\cdots,X_n,Y_n)=1|(0,1))$$

$$= \sum_{\substack{\sum_{T_1}\omega_i X_i\geq r\ ,\ \sum_{T_2}\omega_i^* Y_i\geq s \\ 1\leq k_1\leq n-1,2\leq k_2\leq n}}\sum \sum_{a=a_L}^{a_U} r_n^{01}(k_1,k_2,a)p_{11}^{a}p_{10}^{k_1-a}p_{01}^{k_2-a-1}p_{00}^{n-k_1-k_2+a},$$

$$\mathrm{P}(\phi(X_1,Y_1,\ \cdot\ s,X_n,Y_n)=0|(0,0))$$

$$= \sum_{\substack{\sum_{T_1}\omega_i X_i\geq r\ ,\ \sum_{T_2}\omega_i^* Y_i\geq s \\ 1\leq k_1,k_2\leq n-1}}\sum \sum_{a=a_L}^{a_U} r_n^{00}(k_1,k_2,a)p_{11}^{a}p_{10}^{k_1-a}p_{01}^{k_2-a}p_{00}^{n-k_1-k_2+a-1},$$

where $a_L = \max(0, n-k_1-k_2), a_U = \min(n-k_1, n-k_2)$, k_1, k_2 and a are mentioned for equation (2), T_1 (T_2) extends summation over all k_1 (k_2) combination of $\{1,\cdots,n\}$ and also $r_n^{11}(k_1,k_2,a), r_n^{10}(k_1,k_2,a), r_n^{01}(k_1,k_2,a)$ and $r_n^{00}(k_1,k_2,a)$ are the number of path sets of the system including k_1 working subcomponents from A_i, k_2 working subcomponents from B_i, and a total of a failed component which respectively symbolize the situation of both two subcomponents work, of first subcomponent works and second fails, of first subcomponent fails and second works and both two subcomponents fail.

4 Reliability analysis

Let $(T_1^A, T_1^B),\cdots,(T_n^A, T_n^B)$ represent the lifetimes vectors of n independent and identical components $(A_1,B_1),\cdots,(A_n,B_n)$. As representative of performing the weighted-

(r,s)-out-of-n system, the total weight of the system at time $t(\geq 0)$ can be described by

$$W_n(t) = \sum_{i=1}^{n} \left(\omega_i \mathrm{I}(T_i^A > t) + \omega_i^* \mathrm{I}(T_i^B > t)\right),$$

where $\mathrm{I}(T_i > t)$ is the indicator function (which is 1 if $T_i > t$ and 0 if $T_i \leq t$). Suppose that for $i = 1, \cdots, n$, the marginal survival function of i-th first subcomponent is $\bar{F}_A(t) = \mathrm{P}(T_i^A > t)$ and the marginal survival function of i-th second subcomponent is $\bar{F}_B(t) = \mathrm{P}(T_i^B > t)$, then the mean weight of the system at time t is obtained as

$$\begin{aligned}\mathbb{E}[W_n(t)] &= \sum_{i=1}^{n} \left(\omega_i P(T_i^A > t) + \omega_i^* P(T_i^B > t)\right) \\ &= \sum_{i=1}^{n} \left(\omega_i \bar{F}_A(t) + \omega_i^* \bar{F}_B(t)\right).\end{aligned}$$

Now, considering T as the lifetime of the system, then it is defined as

$$T = \inf\left\{t : \sum_{i=1}^{n} \omega_i \mathrm{I}(T_i^A > t) < r \ or \ \sum_{i=1}^{n} \omega_i^* \mathrm{I}(T_i^B > t) < s\right\}.$$

Now, if the joint failure time distribution of the components (A_i, B_i) is

$$F(t_1, t_2) = P(T_i^A \leq t_1, T_i^B \leq t_2), \qquad \forall i = 1, \cdots, n,$$

then

$$\begin{aligned}&F_{00}(t) = \mathrm{P}(T_i^A \leq t, T_i^B \leq t) = F(t,t),\\ &F_{10}(t) = \mathrm{P}(T_i^A > t, T_i^B \leq t) = F_B(t) - F(t,t),\\ &F_{01}(t) = \mathrm{P}(T_i^A \leq t, T_i^B > t) = F_A(t) - F(t,t),\\ &F_{11}(t) = \mathrm{P}(T_i^A > t, T_i^B > t) = 1 - F_A(t) - F_B(t) + F(t,t).\end{aligned}$$

So, by substituting the joint probabilities p_{00}, p_{10}, p_{01} and p_{11} in equation (6) by $F_{00}(t), F_{10}(t)$, $F_{01}(t)$ and $F_{11}(t)$, respectively, the survival function of weighted-(r,s)-out-of-n system can be established as following

$$\mathrm{R}(t) = \sum\sum_{(k_1,k_2)\in J} \sum_{a=\max(0,k_1+k_2-n)}^{\min(k_1,k_2)} r_n(k_1,k_2,a) F_{11}^{a}(t) F_{10}^{k_1-a}(t) F_{01}^{k_2-a}(t) F_{00}^{n-k_1-k_2+a}(t), \tag{8}$$

where the notations are mentioned for equation (6) in the previous section.

As one of the important reliability characteristics, the MTTF of this system is computed by

$$\begin{aligned}\mathrm{MTTF} = \mathbb{E}[T] &= \int_0^\infty \mathrm{P}(T > t)\mathrm{d}t \\ &= \sum\sum_{(k_1,k_2)\in J} \sum_{a=\max(0,k_1+k_2-n)}^{\min(k_1,k_2)} r_n(k_1,k_2,a)\end{aligned}$$

$$\times \int_0^\infty F_{11}^{a}(t) F_{10}^{k_1-a}(t) F_{01}^{k_2-a}(t) F_{00}^{n-k_1-k_2+a}(t)\mathrm{d}t. \quad (9)$$

Example 4.1. *Consider a weighted-(r,s)-out-of-n system described in Example 3.1 and also, the following Farlie-Gumbel-Morgenstern (FGM) distribution as the bivariate failure time distribution for the subcomponents $(A_i, B_i), i = 1, \cdots, 5$*

$$F(t_1,t_2) = F_A(t_1)F_B(t_2)[1 + \alpha(1 - F_A(t_1))(1 - F_B(t_2))], \qquad t_1, t_2 > 0, \quad -1 \le \alpha \le 1,$$

where $F_A(t_1) = 1 - \exp(-t_1)$ and $F_B(t_2) = 1 - \exp(-t_2)$. So,

$$F_{00}(t) = (1 - e^{-t})^2[1 + \alpha e^{-2t}], \quad (10)$$
$$F_{10}(t) = (1 - e^{-t}) - (1 - e^{-t})^2[1 + \alpha e^{-2t}], \quad (11)$$
$$F_{01}(t) = (1 - e^{-t}) - (1 - e^{-t})^2[1 + \alpha e^{-2t}], \quad (12)$$
$$F_{11}(t) = 1 - 2(1 - e^{-t}) + (1 - e^{-t})^2[1 + \alpha e^{-2t}]. \quad (13)$$

By substituting equations (10)-(13) *and using the entries of Table 1 in equation* (8)*, the survival function of this system is calculated and its plot is given in Figure 1. For* $\alpha = 0.2, 0.5, 0.8$*, the MTTF of the systems is found to be 0.2283, 0.2320, 0.2359, respectively.*

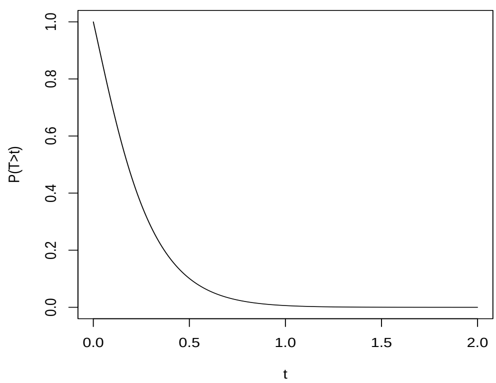

Figure 1: The survival function of weighted-$(7,8)$-out-of-5 system for $\alpha = 0.5$.

5 Conclusions

In many engineering problems, systems with weighted components are useful to deal with such system that components contribute differently to the capacity of the system. A system including n components with their different positive integer weights is known as weighted-k-out-of-n system when it works if and only if the total weight of working components is above a threshold k. In all these considered systems, the components are single elements. But, in some situations, engineers encounter systems consisting of components with two or more subcomponents.

In this paper, an especial setup of weighted-(r, s)-out-of-n system is introduced formed from n independent components each having two dependent subcomponents each with its own positive integer-valued weight. Such a system works if and only if the total weights of the working subcomponents exceed a pre-specified threshold (r, s). The reliability and component importance of this system are studied. The survival function and mean time to failure are presented. Also, some illustrative examples are presented to evaluate the results. As results showed, the system reliability is sensitive to the weights of subcomponents. Also, the system reliability increases when the larger values of working probabilities are assigned to the subcomponents with large weights. This weighted-(r, s)-out-of-n system containing independent components, each having two dependent subcomponents, might be useful in practice.

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